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Effect of strain hardening on the limit slenderness for semi-compact sections

Patrick Studer[1], Andreas Taras[2]

Abstract
The cross-sectional classification forms a key precondition for the safe design of steel sections against local instabilities, as this determines their allowable stress utilization and deformation capacity. This paper focuses on the effect of the stress-strain relationship and of strain hardening on the limiting slenderness values that distinguish between slender or semi-compact (elastic) and compact (plastic) sections. Using test-validated numerical (GMNIA) simulations on short beams, four independent cases were investigated i) elements supported on one edge in pure compression, ii) elements supported on both edges in pure compression, iii) elements of I-sections supported on both edges in pure bending and iv) elements of hollow sections supported on both edges in pure bending. The results of the numerical simulations were analyzed. In particular, the maximum achieved moment was compared with the plastic moment resistance. Thus, a statement could be made on whether the plastic moment resistance could be reached and consequently whether the cross-section should be assigned as a semi-compact (class 3) or compact (class 2) section. From this, limit slenderness values could be determined and correlations to the hardening behavior could be found and integrated into corresponding design formulas.

1. Introduction

Owing to recent technological advances in the steel industry, structural steel with modified properties can nowadays be produced and adapted to the specific requirements of the customer. The adaptable, modified properties include strength, ductility and manufacturing properties, which are of particular importance for the design of constructional steelwork. Through the better utilization of the material this could help to save resources and reduce the environmental impact.

The question of whether or not the plastic (bending) resistance of a section can be utilized is answered, in international design codes, through a system of cross-sectional classification. According to EN 1993-1-1 (CEN – European Committee for Standardization 2005) this plastic resistance can be applied if the cross-section falls into “class 1” or “class 2”. A corresponding classification according to ANSI/AISC 360 (American Institute of Steel Construction 2016) classifies sections as “plastic” or “compact” if they can reach the plastic bending capacity, with the former also allowing for internal moment redistributions. In the remainder of this paper, the Eurocode terminology for classes 1 – 3 will be used, corresponding to AISC classes “plastic”, “compact” and “semi-compact”. Since the classification into the various cross-section classes is

[1] PhD. Student, ETH Zurich, <studer@ibk.baug.ethz.ch>
[2] Professor, ETH Zurich, <taras@ibk.baug.ethz.ch>

based on geometry, yield stress and - in the AISC standard, if not in the Eurocode 3 - the modulus of elasticity, a pronounced hardening behavior is not explicitly considered and differences between steel grades with respect to the post-elastic branch of the stress-strain relationship cannot be exploited by designers. It is however to be expected that beams made of steel with pronounced hardening behavior can be assigned to a different cross-sectional class if the positive effects of hardening behavior can be exploited, as hardening constitutes both a strength increase beyond yield and a "stiffening" of the yielding parts of the section when compared to steels with a yield plateau.

The investigations and findings presented in this paper will thus focus on the influence of the strain-hardening on the cross-sectional classification. In particular, the limiting slenderness values that distinguish between the elastic and plastic design are examined, as there is a considerable change in the utilization of the cross-section. For European standard I-section profiles, the difference between the plastic and elastic moment resistance for strong-axis bending lies between 10 % and 16 %. This shows the great importance of the correct classification of the profiles, as an unfavorable classification leads to lost potential in terms of material savings and economy.

An experimental campaign conducted by the authors (Studer and Taras 2022), where eight four-point bending tests were carried out to determine the moment-rotation behavior of I-sections beams made from different steel grades has given the impetus for the further investigations documented in this paper. Cross sections of class 1 and 3 according to the Eurocode 3 classification system were tested and especially the class 3 cross sections showed significantly different behaviours in dependence of the steel grade. Four different steel grades – S355 J2N, S355M, S460M made to standard provisions of EN 10025, and newly developed steel grade with the commercial designation S355M-*slimfit* (S355M-SF) produced by voestalpine Grobblech GmbH in Linz, Austria, were investigated. These steel grades have varying yield strength ratios, ranging from f_u/f_y = 1.09 (S460M) to 1.67 (S355M-SF). For reference the stress-strain curves of the four steel grades can be seen in Figure 1.

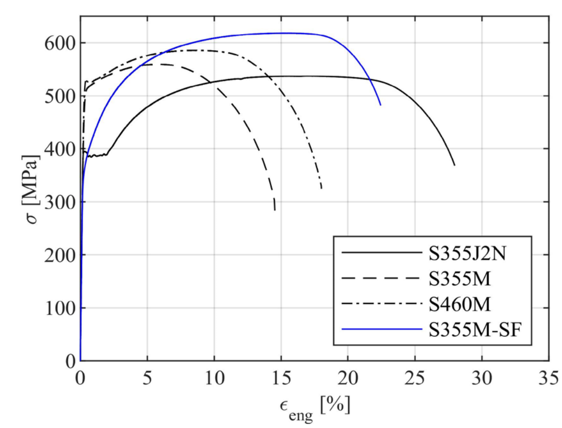


Figure 1: Stress-strain curves of the different analysed steel grades

Geometrically and materially nonlinear (GMNIA) FEM simulations were carried out to systematically investigate the influence of the strain-hardening characteristics on the cross-section classification. The findings are compared with current normative provisions and suggestions are made for taking the hardening behavior in the determination of the cross-section class into account.

2. Cross-sectional classification according to international standards

2.1 Introduction

Most structural sections can be idealized as being comprised of individual flat plate elements. Plate elements are susceptible to local buckling when subjected to compressive stresses or strains. For a utilization of the plastic bending capacity, the plate elements are required to achieve substantial deformations once they have yielded in compression before local buckling occurs, in order for moment redistribution to take place. To prevent premature local buckling, slenderness limits for the plate elements in members have thus been established.

In the following, the procedure for the cross-sectional classification according to the European standard EN 1993-1-1 (CEN – European Committee for Standardization 2005) and the American standard ANSI/AISC 360 (American Institute of Steel Construction 2016) are compared with each other. The differences between the two standards can be divided into the following categories:

- Classes and transition points
- Support and loading conditions
- Slenderness definition
- Material dependency
- Limiting slenderness values

The above listed differences are going to be discussed in more detail in the following subchapters.

2.2 Classes and transition points

Eurocode 3 differentiates between four classes (1, 2, 3, 4) while ANSI/AISC 360 divides the cross-section elements into three different categories (compact, noncompact, slender). While in Eurocode 3 the resistance of the cross-section is determined by the most unfavorable cross-sectional part, the resistance in ANSI/AISC 360 considers the combination of the individual classes of the different cross-section parts. For example, if an I-section is determined to have a class 3 web (decisive) and a class 1 flange according to Eurocode 3, the section would be classified as class 3, just as would be the case for a section with class 3 flange and class 1 web. This is not the case according to ANSI/AISC 360 where an I-section with a slender web and a compact flange has not the same resistance as an I-section with the same slender web and a noncompact flange.

This more detailed and graduated subdivision comes mainly from the fact that there is a greater variety of standardized profiles in America. The geometry of European standard profiles, however, is less variable and the dimensions of the web and the flanges are kind of coordinated so that some of the listed combinations from ANSI/AISC 360 are not even occurring.

In Eurocode 3, the plastic resistance of a section can be applied if the most unfavorable part falls into class 1 or class 2. Otherwise, only the elastic or, if the section is very slender, a reduced elastic resistance can be used. There is a sharp transition between plastic and elastic resistance at a certain slenderness value. The plastic resistance can be applied according to the American standard ANSI/AISC 360 if the flange and the web are classified as compact. According to ANSI/AISC 360, the resistance does not immediately drop to the elastic resistance in the case of slimmer cross-section parts (noncompact, slender); instead, there is a gradual decrease in the resistance. A similar approach was recently implemented in an Annex of the draft version of Eurocode 3 (*FprEN 1993-*

1-1 (CEN – European Committee for Standardization 2022a)), based on results of the research projects SEMI-COMP and SEMI-COMP+ (Knobloch et al. 2020; Lechner et al. 2008)).

2.3 Slenderness definition
There are differences in the definition of the slenderness between European standard Eurocode 3 and the American standard ANSI/AISC 360. Either the mid-thickness width, the clear width or the flat width are divided by the respective thickness to get the slenderness of the element. In the following investigations, only the flat width will be used, as it is done in Eurocode 3. Hence, a conversion factor between flat and mid-thickness width of 0.8 ($c_{flat}/c_{mid\text{-}thickness}$), which is proposed by *Kettler* (Kettler 2008) will be used, for the conversion from one to the other standard. This conversion factor is the outcome of the investigation of rolled European standard H- and I-sections.

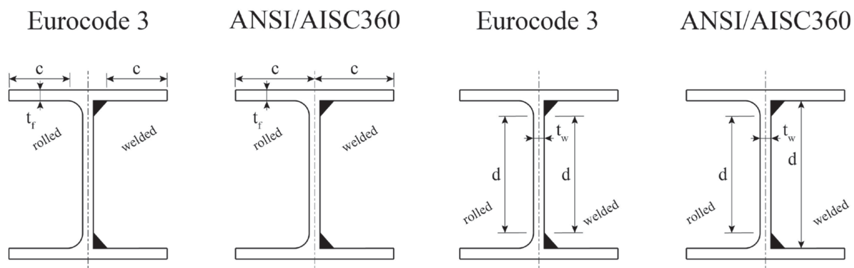


Figure 2: Different slenderness definitions according to Eurocode 3 and ANSI/AISC 360

In addition to the slenderness of the individual cross-section parts, their support and loading conditions are also of great importance.

2.4 Support and loading conditions
Depending on the support and loading conditions the limiting width-to-thickness ratios for the allocation into the different classes are tabulated in the various standards. The American standard deals with many more individual cases than the European standard, where some individual cases are combined into one. For example, the American standard differentiates between the web of an I-section and the web of a rectangular HSS, whereas the European standard considers this to be the same case.

While in Eurocode 3 the loading condition of the individual element is looked at, in the American standard the load case of the entire member is decisive for the classification. The flange of an I-section in pure bending for example, would be categorized according to ANSI/AISC 360 as "member subject to flexure", while according to Eurocode 3 it would be a cross-section element "loaded in pure compression"

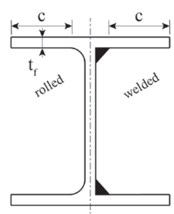
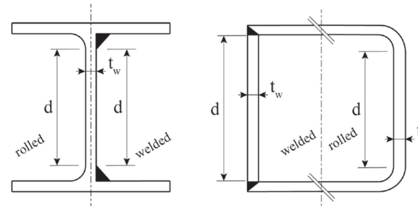
2.5 Material dependency
The classification into the various cross-section classes is based primarily on geometry, but also on basic material parameters. In Eurocode 3 only the yield stress of the material is considered while in the American standard additionally the modulus of elasticity is included.

In the European standard, the yield stress is included using the non-dimensional parameter ε, which is defined as $\varepsilon = \sqrt{235/f_y}$. The American standard directly uses $\sqrt{E/f_y}$ as material dependent parameter, included in the limiting slenderness values.

2.6 Summary

Different limiting slenderness values of selected cases are summarized in Table 1. For better comparability the loading conditions are according to *FprEN 1993-1-1* (CEN – European Committee for Standardization 2022a) meaning that the loading condition of the individual element and not of the entire member is looked at. Additionally, the slenderness definition of *FprEN 1993-1-1* is used, meaning that for the flange the flat width is divided by the thickness. This is done for a steel with a yield strength of 355 MPa and a modulus of elasticity of 210 GPa.

Table 1: Compilation of limiting slenderness values of selected cases for f_y = 355 MPa and E = 210 GPa

	Cross-section elements supported on one edge	Cross-section elements supported on both edges		
				
Eurocode 3				
Class	Loaded in pure compression	Loaded in pure bending		Loaded in pure compression
1	$c/t_f < 9\varepsilon = 7.3$	$d/t_w < 72\varepsilon = 58.6$		$d/t_w < 28\varepsilon = 22.8$
2	$c/t_f < 10\varepsilon = 8.1$	$d/t_w < 83\varepsilon = 67.5$		$d/t_w < 34\varepsilon = 27.7$
3	$c/t_f < 14\varepsilon = 11.4$	$d/t_w < 121\varepsilon = 98.4$		$d/t_w < 38\varepsilon = 30.9$
ANSI / AISC 360				
Class	Loaded in pure compression (Members subject to flexure)	Loaded in pure bending (Members subject to flexure)		Loaded in pure compression (Members subject to flexure)
		I-shaped	rectangular HSS	
compact	$c/t_f < 0.8$[1]$\cdot 0.38(E/f_y)^{0.5}$ = 7.4	$d/t_w < 3.76(E/f_y)^{0.5}$ = 91.4	$d/t_w < 2.42(E/f_y)^{0.5}$ = 58.9	$d/t_w < 1.12(E/f_y)^{0.5}$ = 27.2
noncompact	$c/t_f < 0.8$[1]$\cdot 1.0(E/f_y)^{0.5}$ = 19.5	$d/t_w < 5.70(E/f_y)^{0.5}$ = 138.6	$d/t_w < 5.70(E/f_y)^{0.5}$ = 138.6	$d/t_w < 1.40(E/f_y)^{0.5}$ = 34.1

1. cflat/cmidthickness = 0.8

As can be seen in Table 1, there are some similarities between the limiting slenderness values of the two different standards. The limiting slenderness values defining class 1, or compact cross-sections, are of a very similar order of magnitude. With a greater discrepancy but still of a comparable order of magnitude are the limiting slenderness values for class 3 and non-compact cross-sections. Overall, a class 1 cross-section according to Eurocode 3 can be compared to an American compact section and a class 3 cross-section to a noncompact one, but with a slightly poorer match.

An I-section beam loaded in major-axis ("in-plane") bending is used to illustrate the difference between the two mentioned standards. According to Eurocode 3, the flange is loaded in pure compression, and the web in pure bending. The moment resistance normalized by the elastic moment resistance is shown in Figure 3 as a function of the respective slenderness. In Figure 3a, the slenderness of the flange is changed while the one of the web is constant (class 1/compact) while in Figure 3b the slenderness of the web is varied while the slenderness of the flange is held constant (class 1/compact). The curve according to the American standard is shown in black and the European pendant is shown in blue. Additionally, the limiting slenderness values are shown in the respective color as dashed lines. Below the diagrams, the cross-sectional classes are stated.

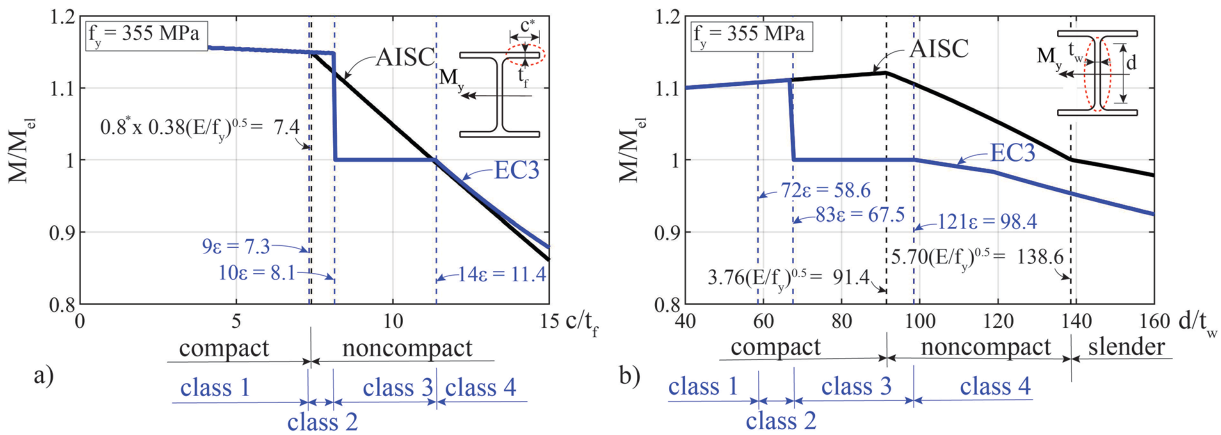


Figure 3: a) classification of a cross-section element supported on one edge in pure compression according to Eurocode 3 respectively ANSI/AISC 360 (*cflat/cmidthickness = 0.8) and b) classification of a cross-section element supported on both edges in pure bending according to Eurocode 3 respectively ANSI/AISC 360

As can be seen in Figure 3a, the limiting slenderness value between compact and noncompact sections according to ANSI/AISC 360 is quite similar to the limiting slenderness value between class 1 and class 2 according to Eurocode 3. While the plastic moment resistance is applicable until the end of the class 2 cross-section range, it begins to decrease at the limiting slenderness value between compact and noncompact sections. While it drops down sharp to the elastic moment resistance according to the European standard (discounting, for better comparability, the SEMI-COMP methodology of the draft code FprEN1993-1-1:2022), there is a continuous decrease according to the American standard. Figure 3b shows some bigger differences between the two standards. The plastic moment resistance can be much longer applied according to ANSI/AISC 360 compared to Eurocode 3. There is again a sharp drop between cross-section class 2 and 3 while a continuous decrease can be seen after exceeding the compact cross-section class.

3. Material and Methods

Based on the outcome of experimental investigations by the authors (Studer and Taras 2022), the study presented in this paper aimed at identifying and quantifying the effects of different stress-strain relationships of various steel grades on the cross-sectional classification system. In experimental tests, especially the cross-sections classified as being in class 3 according to Eurocode 3 showed a very different behavior, depending on the steel grade, with some sections classified as class 3 even reaching and exceeding the plastic bending resistance. This fact has initiated the in-depth investigation of the transition between cross-section class 2 and 3 discussed in this paper. The following sub-sections illustrate the employed methodology for this study.

3.1 Procedure

The investigations on the transition between cross-sectional class 2 and 3 was carried out using numerical simulations. Therefore, a parametrized model was created, with which numerous simulations of beams with various geometries and material properties could be created. To comply with the logic of the cross-section classification according to Eurocode 3, whole I-section and hollow-section beams were simulated. As in the Eurocode classification, cross-section elements supported on one or both edges were considered and loaded by different stress distributions. Since all considered elements were beams and were thus subjected to a bending moment about their major axis, the following cases were covered:

- Flange of I-section beam: element supported on one edge in pure compression ①
- Web of I-section beam: element supported on both edges in pure bending ②
- Flange of hollow-section beam: element supported on both edges in pure compression ③
- Web of hollow-section beam: element supported on both edges in pure bending ②/④

According to the European standard, a total of three different cases are covered, whereby case 2 is represented twice. However, the American standard differentiates also between the web of I-section and hollow section beams and thus four individual cases are present. Therefore, in the following, these four different cases will be considered separately and, where appropriate, compared with each other.

In order to exclude the possibility of the adjacent cross-section parts influencing the results, the latter were modelled and selected in such a way that they would not be decisive for the local buckling behaviour: they themselves were of “stocky dimensions” (class 1 or 2) in terms of their proneness to buckling. In addition, an imperfection was applied to the cross-sectional part to be examined, while the rest of the cross-section has been modelled without imperfection. The general FE analysis software ABAQUS (Abaqus 2018) was used for the numerical simulation. The beams were modelled using shell elements and meshed by S4R elements, which are 4-node, quadrilateral, stress/displacement shell elements with linear interpolation function and reduced integration.

3.2 Considered material models

In order to standardize and simplify the stress-strain relationships to be considered in the study while maintaining a sufficient degree of generalization in the outcomes, a trilinear strain-hardening material model was used in the present study, thereby aiming at approximation the behavior of steels without a pronounced yield plateau. This model generally underestimates the stresses in the strain-hardening branch, because the real stress-strain behavior in the strain-hardening branch typically follows a convex shape, resulting in a slight underestimation at the same elongation. It can therefore be assumed to be a conservative material model for steel without a yield plateau. To investigate the influence of different strain-hardening behaviors a wide range of yield-strength ratios has been studied. In addition to the yield-strength ratio, also the elongation ε_u, when the tensile strength f_u is reached, was varied. The material parameters can be taken from Table 2.

Table 2: Material parameters used for the numerical simulations

f_y [MPa]	f_u/f_y [-]	ε_u [%]				E [MPa]
235 355 460	1.10 1.25 1.50 1.75	2.5	5.0	10.0	15.0	210’000

The listed material parameters in Table 2 resulting in 16 different strain-hardening characteristics. After reaching the tensile strength f_u at elongation ε_u the stress was held constant. The stress vs. strain diagram for these “artificial” steel grades can be seen in Figure 4.

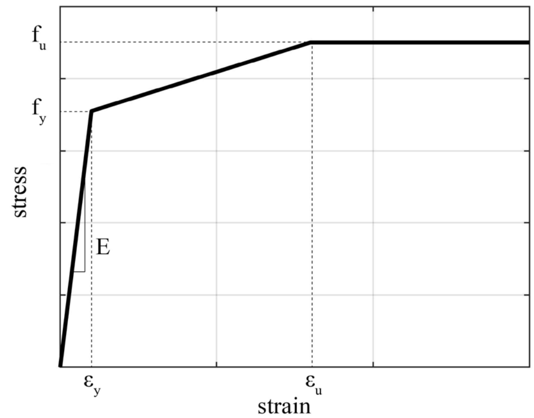


Figure 4: Engineering stress-strain diagram for the artificial steel-grades

3.3 Geometry, Boundary conditions and Imperfections

In order to simulate geometries that are as close to reality as possible, the cross-sections of common European I- and H-sections were studied along with hollow sections. Based on these obtained geometry ratios and their distribution, profile dimensions were chosen to achieve a wide range of slenderness ratios of the corresponding cross-section. For the investigations on the flange slenderness, the variation of the width for the simulated geometries resulted in cross-section classes 1 to 4, whereas the web was chosen in a way to fall into class 1 or class 2, in order to have only a small influence on the outcome of the simulations in terms of proneness to instability. Since the slenderness limits according to Eurocode 3 neglect the interaction between the plate elements forming the section, both flanges are connected to the web, but the rotation around their longitudinal axis (UR1) is free. Both end faces are connected to a control point with an MPC constraint. A beam-type MPC constraint was chosen, to define a rigid beam connection to constrain the displacement and rotation of each slave node to the displacement and rotation of the control point. Via these control points, the forces and moments are applied, and the boundary conditions are also defined at these points. In Figure 5a+b the schematic connections and control points including the corresponding boundary conditions are shown for the I-section beam. The same conditions were used for the hollow sections.

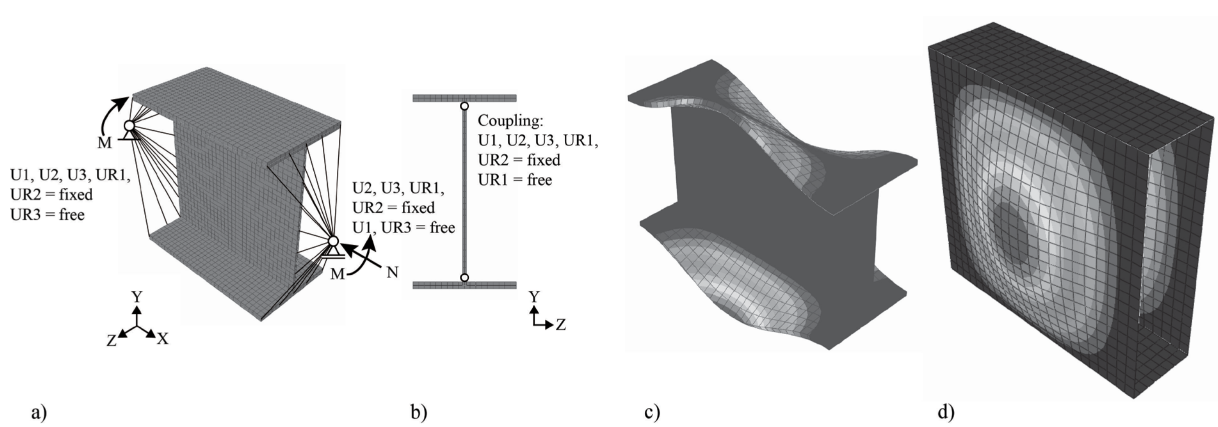


Figure 5: Boundary conditions and imperfections for the FE analysis: a) boundary conditions and schematic connections, b) coupling conditions between flanges and web, first eigenform for the investigation of c) elements supported on one edge in pure compression and d) elements supported on both edges in pure bending

The length of the beams was chosen to be no longer than needed in order for the elastic critical buckling stress obtained from the LBA to approximate with minimal error the theoretical (minimal) value. In order to obtain approximately the theoretical elastic critical buckling stress the following lengths of the beams were thus chosen, based on *Timoshenko* (Timoshenko 1970) and *Yu and Schafer* (Yu and Schafer 2007):

- Supported on one edge, loaded in pure compression: l = 2.5 b
- Supported on both edges, loaded in pure bending: l = 1.0 h
- Supported on both edges, loaded in pure compression: l = 2.5 h

The first eigenform was applied as an initial imperfection shape to the GMNIA simulation and scaled using a maximum amplitude of 0.5 b/200 for the case when the cross-section part held on one edge and b/200 resp. h/200 for the case when the cross-section part held on both edges, following recommendations given in *EN 1993-1-5 Annex C* (CEN – European Committee for Standardization 2010) and *prEN1993-1-14* (CEN – European Committee for Standardization 2022b). In Figure 5c the first eigenform can be seen for the case when investigating the flange of the I-section beam and in Figure 5d the same can be seen for the investigation of the web of a hollow section beam.

4. Results

4.1 Evaluation Procedure

In order to investigate the transition from cross-sectional class 2 to 3, which describes whether the plastic moment resistance is reached or not, three main steps are involved: 1) the maximum reached moment based on the numerical simulation is determined, 2) this maximum reached moment is normalized by the plastic moment resistance and plotted against the slenderness of the cross-section for different yield-strength ratios and the slenderness intersecting $M_u/M_{pl} = 1$ is determined and 3) this determined slenderness is plotted against the yield-strength ratio f_u/f_y. Figure 6 illustrates the mentioned steps. These steps can be repeated not only for different yield strength ratios but also for materials with different elongations ε_u.

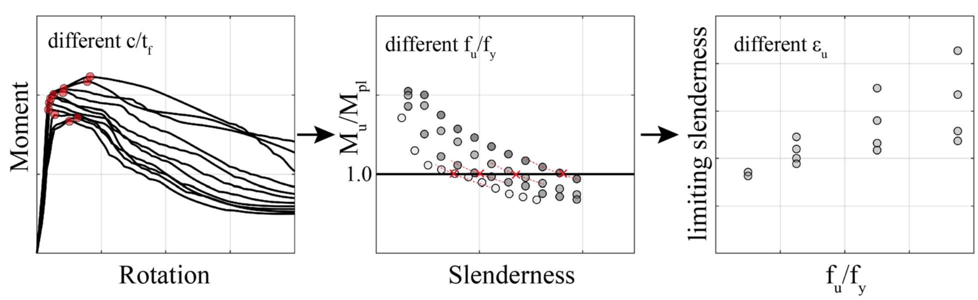


Figure 6: Main steps for the evaluation of yield-strength ratio dependent limiting slenderness values for the plastic design

With the aforementioned procedure, it is possible to obtain limiting slenderness values for the plastic design which are dependent on the strain-hardening behavior.

4.2 Limiting slenderness values

The obtained limiting slenderness values for the four different cases are depicted in Figure 7a-d. The different colors stand for the different elongations ε_u and additionally the limiting slenderness values from the standards *Eurocode 3* (blue) and *AISC* (black) are shown. The three columns represent the three investigated steel grades S235, S355 and S460.

Figure 7a shows the yield-strength ratio dependent limiting slenderness values for elements supported on one longitudinal edge, loaded in pure compression. The limiting slenderness values are increasing with the yield-strength ratio and are higher with lower elongation ε_u. The difference between the different elongations ε_u becomes more pronounced with increasing yield strength ratio. Compared with the limiting slenderness values from the two standards, one can see, that the determined limiting slenderness values are considerable higher and especially with increasing yield-strength ratio the ones from the standards are quite conservative. With increasing yield strength, the limiting slenderness values are decreasing, which is well in line with today's standards.

The same trend of increasing limiting slenderness values with increasing yield strength ratio and decreasing elongation ε_u can also be seen for the three other cases (see Figure 7b-c). For those cases, however, the limiting slenderness values according to the standards are more of an average value of the obtained limiting slenderness values from the numerical simulations. Some of the obtained points are below the limits from the standards, especially for small yield-strength ratios and high elongations ε_u, while others are above the limits from the standards.

5. Design recommendations

Based on the findings illustrated and discussed above, new recommendations for the limiting slenderness values for the plastic design are derived. Therefore, the transition points between cross-sectional class 2 and 3 are approximated by continuous curves. The investigations have shown that the curves can be approximated well by a linear function, which depends directly on the yield-strength ratio, as can be seen in Eq. (1).

$$c/t_{f,lim,2\_3}\ resp.\ d/t_{w,lim,2\_3} = Y_1 \frac{f_u}{f_y} + Y_2 \qquad (1)$$

The two coefficients Y_1 and Y_2 were determined by an in-depth analysis of their dependence on the elongation ε_u and the dependency on the yield strength f_y, which is expressed as $\varepsilon = \sqrt{235/f_y}$. Thereby, it was finally observed that the coefficient Y_2 is nearly constant regarding the elongation ε_u and has no big variability, has, however, a dependence on the yield strength itself. The coefficient Y_1 was proposed to be defined as shown in Eq. (2) for the case of elements supported on one or two edges in pure compression and as shown in Eq.(3) for elements supported on both edges in pure bending.

$Y_1 = \frac{1}{\varepsilon}(a\ \varepsilon_u^b + c)$	For elements in pure compression	(2)
$Y_1 = \varepsilon(a\ \varepsilon_u^b + c)$	For elements in pure bending	(3)

Thereby, three new unknown parameters were introduced and, in turn, calibrated against the retrieved numerical data and results. They were determined by a least-squares fitting procedure. Note that ε is the material dependent parameter which depends on the yield strength f_y and ε_u is the elongation at which the tensile strength f_u is reached. The material dependency of Y_2 can be seen in Eq.(4).

$$Y_2 = \frac{d f_y}{235} + e \tag{4}$$

The parameters *d* and *e* were again determined by a least-squares fitting procedure.

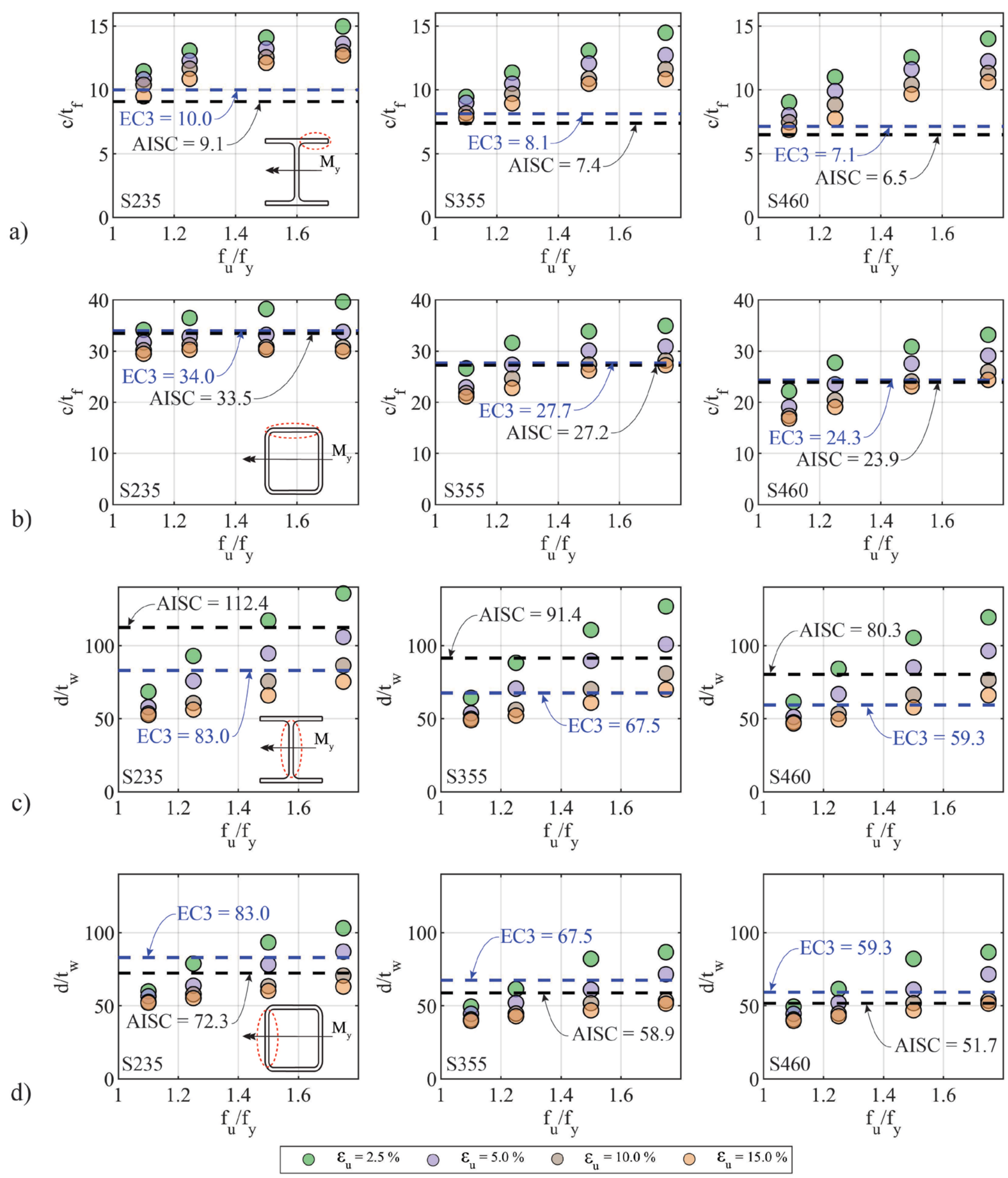
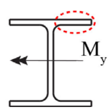
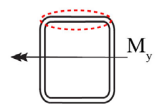
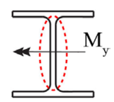
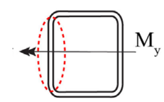


Figure 7: Limiting slenderness values as a function of the yield-strength ratio and elongation ε_u; a) elements supported on one edge in pure compression, b) elements supported on both edges in pure compression, c) elements supported on both edges in pure bending (I-sections) and d) elements supported on both edges in pure bending (hollow sections)

The procedure was iterative, as two parameters had to be optimised at the same time. Parameter Y_2 was first varied for one steel grade and then applied to the different steel grades using the given relationship from Eq.(4). Using this Y_2, the optimum Y_1 could now be found by minimising the error to the given relation from Eq. (2) or Eq.(3). Through this *physically informed* fitting procedure the parameters Y_1 and Y_2 could be found which optimally follow the given relationship. The best fitting pair of the two parameters could be determined by minimising the unsafe sided results which occur between the true (from numerical simulations) and the calculated design limiting slenderness values.

In summary, the parameters in Table 3 could be obtained for the calculation of coefficient Y_1 and Y_2 for the four different supporting and loading conditions.

Table 3: Parameters for the calculation of Y_1 and Y_2

Case	Elements supported on one edge in compression	Elements supported on both edges in compression	Elements supported on both edges in bending	Elements supported on both edges in bending
Parameter				
a	-1.47	10.74	90.07	60.55
b	0.28	-0.50	-0.64	-0.42
c	7.53	4.89	54.02	15.76
d	-5.00	-14.00	27.00	5.00
e	10.00	33.00	-64.00	-1.00

The chosen linear function was found to reflect the behaviour of the studied cases very well. In order to illustrate this further, Figure 8 compares the true transition c/t ratio (obtained from the numerical simulations) with the transition according to the proposed design. In order to see whether the design values are on the safe side, i.e. predict lower values compared to the actual limit values, the residual values are plotted in Figure 8. The residual is defined as the “true” value minus the prediction and should, in the safe case, result in a positive value.

A distinction is again made between the four cases i) cross-section elements supported on one edge in pure compression (see Figure 8a), ii) cross-section elements supported on both edges in pure compression (see Figure 8b), iii) cross-section elements of I-section beams supported on both edges in pure bending (see Figure 8c) and iv) cross-section elements of hollow section beams supported on both edges in pure bending (see Figure 8d).

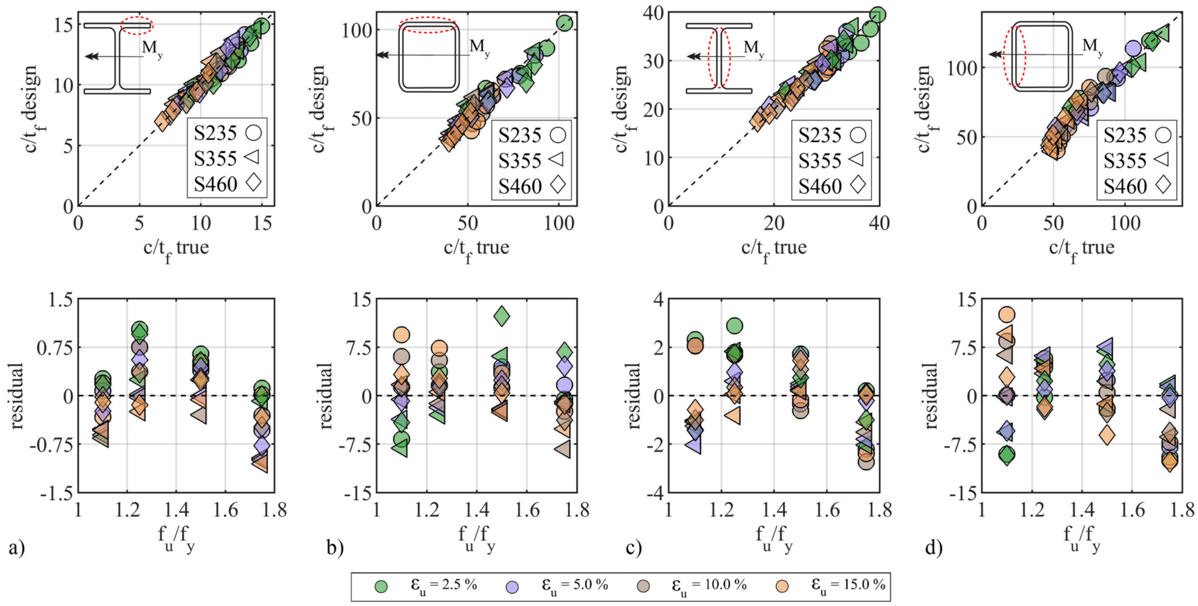


Figure 8: Comparison between true transition slenderness values and proposed design slenderness values for: a) elements supported on one edge in pure compression, b) elements supported on both edges in pure compression, c) elements supported on both edges in pure bending (I-sections) and d) elements supported on both edges in pure bending (hollow sections)

As can be seen in the upper half of Figure 8, the points are arranged around the bisector, which means that the determination of the transition point with the proposed procedure results in approximately identical values as the true values obtained from the numerical simulations. The residuals are mostly positive, which means that the proposed design procedure predicts safe sided values. There are some negative residuals for all four cases but compared to the value itself, there is only a deviation in the lower double-digit percentage range. For all cases, some larger deviations between the true values and the design values can be detected. This is mostly due to the fact that the design equation was developed as a lower-bound approximation of observed results, which vary significantly as an inherent effect of the design procedure with “isolated plates”. This is seen as acceptable for design purposes, as the proposal still significantly improves on the current approach that does not explicitly account for yield strength ratios and hardening behaviour.

6. Discussion

6.1 Comparison with the existing limiting slenderness values

As already shown in Figure 7a-d, there is not only a difference between the slenderness values obtained from the numerical simulations and the standards but also between the two different standards itself.

For flanges of I-sections loaded by in-plane (respectively strong-axis) bending, current ANSI/AISC 360 and Eurocode 3 values appear to be appropriate and safe-sided for f_u/f_y values of above 1.1 and any value of ε_u, which would be typical for any mild steel typically found on the market. For flanges of hollow-sections loaded by in-plane bending, somewhat higher values of f_u/f_y, of around 1.6, appear to be implied by the current classification limit. For webs of I-sections, the study presented in this paper indicates that the current Eurocode 3 limits are somehow an average value over the investigated hardening characteristics, while the limits according to

ANSI/AISC 360 tend to be at the upper end of the obtained limiting slenderness values. For webs of hollow sections, both standards are in the same order of magnitude and are somehow an average value over the investigated steel grades. It should however not be forgotten that the webs of sections are commonly not the determining factor in the overall plastic moment capacity of beam sections: a lack of safety of current ANSI/AISC 360 or Eurocode 3 rules should thus not directly be inferred from the above results.

Due to the lack of distinction between I-sections and hollow sections, the biggest difference between the two standards can be seen for the case of webs of I-sections. It seems, that the limiting slenderness values for cross-section elements supported on both edges (webs) in pure bending according to Eurocode 3 are more appropriate for hollow sections. This topic is dealt with in more detail in the following subsection.

6.2 Subdivision of the webs of different profile types

The investigations have shown that the differentiation between the web of an I-shaped section and the web of a hollow section, as is done in the AISC standard, is useful. The trend, that for I-shaped sections the plastic resistance can be used for a wider range of slenderness values could be confirmed. This is especially the case for steel grades with higher elongations ε_u as can be seen in Figure 9. The limiting slenderness values for I-sections are shown as circles and continuous lines while the ones for hollow sections are depicted as squares and dashed lines.

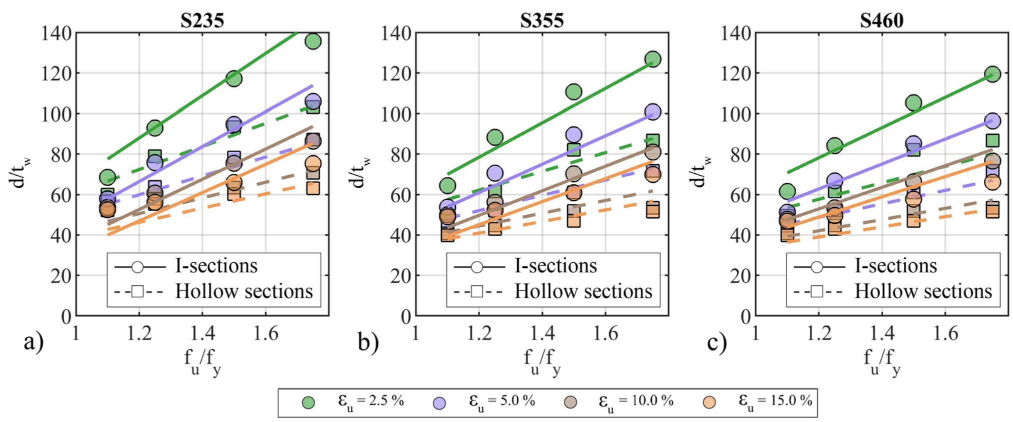


Figure 9: Comparison of the limiting slenderness for webs of I- and hollow sections for different steel grades: a) S235, b) S355 and c) S460

7. Summary, Conclusion and Outlook

The study described in this paper focused on gathering a better understanding of the significance of strain hardening and the yield strength ratio f_u/f_y on the transition plate slenderness between cross-sectional classes 2 and 3, which for beams defines whether the plastic moment resistance can be exploited or not. Numerical simulations were carried out on short beams, where the first eigenform from an LBA simulation was used as initial imperfection for a GMNIA simulation. The results of the numerical simulations were analysed; in particular, the maximum moment was compared with the plastic moment resistance. Thus, a statement could be made on whether the plastic moment resistance could be achieved or not.

Based on these results, the limiting slenderness value for plastic design could be determined in dependence of the yield-strength ratio f_u:f_y as well as on the elongation ε_u. It was confirmed that a high yield strength ratio has a positive effect on the limiting slenderness value, i.e. this boundary is located at higher slenderness ratios. The opposite is true for the elongation ε_u, where lower values have a positive influence on the limiting slenderness. This is intuitive, since the higher the yield strength ratio and the lower the elongation ε_u, the steeper the increase in the hardening range is, resulting in higher stresses with the same deformation. Based on these findings, methods were developed to quantify these effects in an improved classification procedure.

The investigation on different types of cross-sections (I-sections and hollow sections) has shown that the differences between these section shapes, even when individual plates within this section have the same geometric and material characteristics, can have a significant influence on the "correct" classification. The case of cross-section elements supported on both edges in pure bending, which is present in both types of cross-sections, was found to be best represented by different limiting slenderness values in dependence of the cross-section type.

The developed proposal for an improved definition of the limit slenderness value between classes 2 and 3 for I- and hollow sections takes into account the key parameters of a monotonic stress-strain curve without pronounced yield plateau by means of simple formulae for the resulting c/t-values. The formulations are shown to be sensitive to these parameters and reflect the results from the numerical investigations with good accuracy. Independent of the considered standard (Eurocode 3 or ANSI/AISC 360), the consideration of further material parameters, such as the yield-strength ratio and the elongation ε_u, can help to refine the classification and thus, contribute to a better and more realistic cross-section utilization. This represents a step forward in enhancing the sophistication and efficiency of structural steel design through advanced analysis.

In future studies, it is planned to advance the first proposals in this paper further, including more realistic and varied types of stress-strain relationships (among them, steels with pronounced yield plateau) and to expand the study on the additional limit values, such as those separating semi-compact from elastic sections.

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