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SIPC – A practical geometric nonlinear analysis method for the design of metal structures

Ronald D. Ziemian[1], Constance W. Ziemian[2]

Abstract
Geometric nonlinear or second-order effects are the changes to forces, moments, and displacements that result when solving the equations of equilibrium on the actual deformed shape rather than the undeformed shape of a structure. For most civil engineering structures constructed of steel or aluminum and designed for serviceability, these additional nonlinear effects are significant enough to warrant consideration when designing for strength, but are often not substantial enough to require an extremely accurate numerical solution scheme. With this in mind, this paper proposes the use of an approximate second-order elastic solution method that utilizes only two linear analyses within a single load increment. Further contributing to the method's efficiency is its use of analysis results from the serviceability design process in the initial or predictor step, thereby requiring only one linear corrector analysis per load combination investigated. Through comparisons with a more exact solution scheme, twenty-two steel benchmark frames have been used to demonstrate the method's ability to maintain sufficient accuracy while significantly improving computational efficiency. Consideration is given to the advantages and limitations of the method, leading to a more general discussion regarding frame sensitivity to second-order effects.

1. Introduction
Civil engineering structures are typically designed using elastic analyses to assess both serviceability and strength requirements. With regard to serviceability, the deflections are small thereby permitting analysis results for individual load types (dead, live, wind, etc.) to be factored and summed as needed to assess specific design requirements. With regard to strength requirements, however, the deflections are large enough that the opportunity to employ superposition is often not permitted. As a result, separate geometric nonlinear analyses are needed for the large number, often on the order of hundreds, of load combinations that may require consideration during preliminary and final design processes.

With this in mind, it is important for the engineer to maintain an effective balance between the needs of the current design problem and the accuracy, required modeling effort, and computational

[1] Professor, Bucknell University, <ziemian@bucknell.edu>
[2] Professor, Bucknell University, <cziemian@bucknell.edu>

expense of the relevant geometric nonlinear analysis methods. The available analysis options range from a rigorous 'nearly exact' geometric nonlinear analysis at one extreme, to the approximation of nonlinear effects through the combined use of linear analyses and moment amplification factors, at the other extreme. Although the latter is often an efficient choice, its practicality usually lends itself only to structural systems with orthogonal framing of uniform heights that are continuous across all bays. In addition, the use of moment amplification approaches is limited in some codes and unavailable in others (ADM 2020).

This paper is based upon a detailed study completed and presented by Ziemian and Ziemian (2020). Using a suite of benchmark problems for assessment, the proposed approximate second-order elastic analysis algorithm, which is based on the finite element method, meets the following goals:

- Efficient for evaluating a large number of load combinations in design and/or rapidly analyzing preliminary designs
- Applicable to a full range of geometries
- Utilizes the results of analyses completed to assess serviceability conditions
- Performs with an acceptable level of accuracy for the design of 'typical' steel structures, i.e. having nonlinear responses that require consideration, but not at levels that warrant a rigorous "nearly exact" analysis

2. SIPC Method

In a geometric nonlinear analysis, the total load on the structure is typically applied incrementally, with the geometry and internal force distribution updated at the end of each load increment (McGuire et al. 2000). The more accurate the representation of the nonlinear relationship between load increment and the resulting increment in deflection, the better the piecewise fit will approximate the exact equilibrium solution. For highly nonlinear behavior, several iterations are often performed within a series of reduced load increments, thereby resulting in an accurate result but at significant computational cost.

The method presented and assessed in this study is based on a strategy equivalent to solving systems of differential equations using a mid-point Runge-Kutta method – often referred to as a predictor-corrector solution scheme. As shown in Fig. 1, the displacement $\{\Delta_i\}$ for factored load combination i is determined without iteration, by employing a representative system-stiffness matrix $[\overline{K}_i]$ and solving the linear system of equations for a given applied load $\{P_i\}$

$$[\overline{K}_i]\{\Delta_i\} = \{P_i\} \tag{1}$$

The deformed geometry and corresponding element forces used in computing the system's midpoint stiffness $[\overline{K}_i]$ are approximated as one-half the sum of the factored displacements and element forces corresponding to each load type (i.e. live, dead, wind, etc.) appearing in load combination i. As such, a first-order elastic analysis for all load types within the factor load combinations being considered would first need to be completed.

This single increment predictor-corrector (SIPC) solution scheme can be an efficient second-order RK method that employs only two linear analyses, at two sampling points, within a single load increment. In addition, and as outlined in the next section, the first of these two analyses need only be performed once (during serviceability design).

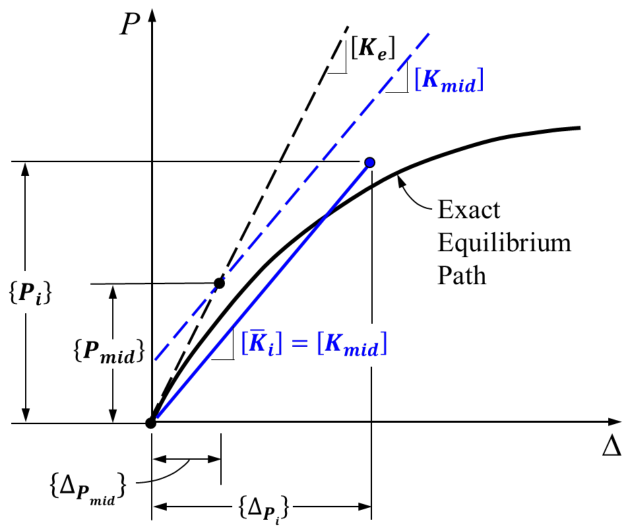


Figure 1: Graphical representation of the single increment predictor-corrector (SIPC) solution scheme

2.1 Use of SIPC method in the design process

The SIPC solution scheme is proposed for use as an approximate method for the geometric nonlinear (second-order elastic) analysis of a structural system. And, it can be used for the analysis and design of any frame geometry as follows.

Design for Serviceability

In assessing the serviceability requirements in the design of a steel or aluminum frame, a linear (first-order elastic) analysis is performed to determine the deflections and member forces within the structural system. This is typically done by performing a single linear analysis for all of the different load types being considered, including dead, live, and wind. Mathematically, this involves solving the governing system of stiffness equations for the unknown displacement vectors that correspond to a given set of unfactored applied load vectors. It is important to note that the system stiffness matrix is not a function of the applied loads and is representative of the undeformed and unloaded geometry, and therefore, the serviceability analysis can be done by solving a single system of linear equations with multiple right-hand sides (Golub and Van Loan 2013).

For example, consider Eq. 2, in which $[K_e]$ is the system's first-order elastic stiffness matrix, $\{P_{load}\}$ are vectors representing the unfactored dead (D), live (L), and wind (W) load on the system, and $\{\Delta_{load}\}$ are vectors signifying the resulting system deflections due to these dead, live, and wind loads.

$$[K_e]\big\{\{\Delta_D\} : \{\Delta_L\} : \{\Delta_W\}\big\} = \big\{\{P_D\} : \{P_L\} : \{P_W\}\big\} \tag{2}$$

The solution for this system of linear equations with multiple right-had sided involves:

a. a one-time decomposition of the stiffness matrix $[K_e]$

$$[K_e] = [L][L]^T \tag{3}$$

where $[L]$ is a lower triangular matrix,

b. a simple forward substitution to solve for temporary vectors $\{\Upsilon_i\}$,

$$[L]\{\{\Upsilon_D\} : \{\Upsilon_L\} : \{\Upsilon_W\}\} = \{\{P_D\} : \{P_L\} : \{P_W\}\} \tag{4}$$

c. a backward substitution to solve for each of the displacement vectors

$$[L]^T\{\{\Delta_D\} : \{\Delta_L\} : \{\Delta_W\}\} = \{\{\Upsilon_D\} : \{\Upsilon_L\} : \{\Upsilon_W\}\} \tag{5}$$

It is noted that the majority of the computational time in this analysis is spent on the one-time decomposition of the system stiffness matrix (Eq. 3). In other words, separate first-order elastic analyses are not required for each load type.

Vectors of element forces and moments $\{\hat{f}\}$, with reference to their local coordinate systems, are computed for each of the load types. For example, such values for a given element would be obtained from the matrix multiplication

$$[\hat{k}_e][\Gamma_o]\{\{\delta_D\} : \{\delta_L\} : \{\delta_W\}\} = \{\{\hat{f}_D\} : \{\hat{f}_L\} : \{\hat{f}_W\}\} \tag{6}$$

in which, $[\hat{k}_e]$ is the element's first-order elastic stiffness matrix relative to its local coordinate system, $[\Gamma_o]$ is the element's global-to-local coordinate transformation matrix based on the system's original undeformed geometry, and $\{\delta\}$ are vectors of element end displacements extracted from the vectors of system deflections $\{\{\Delta_D\} : \{\Delta_L\} : \{\Delta_W\}\}$. These displacement vectors are then used in the design process to perform serviceability checks, and thereby confirm that the members and system are of adequate stiffness. For example, live load beam deflections would be assessed using the information provided in the deflection vectors $\{\delta_L\}$.

Design for Strength

In further designing the structural system for strength, the SIPC method begins with a predictor step that uses the first-order elastic analysis results from the serviceability checks. Specifically, for each factored load combination being considered, superposition (scaling and summing) of the previously determined nodal displacements $\{\Delta_D\}$, $\{\Delta_L\}$, and $\{\Delta_W\}$ and element forces and moments $\{\hat{f}_D\}$, $\{\hat{f}_L\}$, and $\{\hat{f}_W\}$ are used.

For example, for the factored load combination $\alpha D + \beta L + \gamma W$, the strength analysis would proceed as follows.

a. The original geometry of the structural system (i.e. x, y, and z coordinates of each node) is modified to the midpoint geometry by adding 50% of the factored combined displacements. The updated coordinates of the i^{th} node are thus

$$\begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{mid} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_o + 0.5\left(\alpha \begin{Bmatrix} \Delta_{x,i} \\ \Delta_{y,i} \\ \Delta_{z,i} \end{Bmatrix}_D + \beta \begin{Bmatrix} \Delta_{x,i} \\ \Delta_{y,i} \\ \Delta_{z,i} \end{Bmatrix}_L + \gamma \begin{Bmatrix} \Delta_{x,i} \\ \Delta_{y,i} \\ \Delta_{z,i} \end{Bmatrix}_W \right) \tag{7}$$

b. Similarly, element forces and moments for use in forthcoming element geometric stiffness matrices are taken as 50% of the combinations of their factored values given above, noting that it is assumed that all element forces and moments are originally zero (i.e. the system is originally unloaded). The updated element forces and moments in the j^{th} element are

$$\{\hat{f}_j\}_{mid} = [\Gamma_{j,mid}]\,[\Gamma_{j,o}]^T \left\{0.5\left(\alpha\{\hat{f}_j\}_D + \beta\{\hat{f}_j\}_L + \gamma\{\hat{f}_j\}_W\right)\right\} \tag{8}$$

where $[\Gamma_{j,o}]$ is the element's global-to-local coordinate transformation matrix based on the system's original undeformed geometry, and $[\Gamma_{j,mid}]$ is element's global-to-local coordinate transformation matrix based on the system's updated geometry per previous step (a).

c. The system of equilibrium equations for the geometric nonlinear analysis is

$$[K_e + K_g]\{\Delta_{\alpha D+\beta L+\gamma W}\} = \{P_{\alpha D+\beta L+\gamma W}\} \tag{9}$$

where $\{P_{\alpha D+\beta L+\gamma W}\} = \alpha\{P_D\} + \beta\{P_L\} + \gamma\{P_W\}$, and $[K_e + K_g]$ is assembled from the element elastic and geometric stiffness matrices. These matrices are computed for each element with

$$[k_e + k_g] = [\Gamma_{mid}]^T[\hat{k}_e + \hat{k}_g][\Gamma_{mid}] \tag{10}$$

in which the element's global-to-local coordinate transformation matrix $[\Gamma_{mid}]$ is based on the updated geometry defined in previous step (a), $[\hat{k}_e]$ is the element's elastic stiffness matrix, and $[\hat{k}_g]$ is the element's geometric stiffness matrix based on the element forces and moment computed in previous step (b).

d. Once again using an efficient algorithm, such as banded Cholesky decomposition (Golub and Van Loan 2013), the above system of equilibrium equations (Eq. 9) is solved for the system displacements $\{\Delta_{\alpha D+\beta L+\gamma W}\}$ that correspond to the applied loads of the given factored load combination $\{P_{\alpha D+\beta L+\gamma W}\}$. As with Eq. 3, the majority of the computational time in this analysis is spent on this step.

e. The final geometry of the structural system (i.e. x, y, z coordinates of each node) is based on the deflections computed in previous step (d). The updated coordinates for the i^{th} node are

$$\begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{final} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_o + \begin{Bmatrix} \delta_{x,i} \\ \delta_{y,i} \\ \delta_{z,i} \end{Bmatrix}_{\alpha D+\beta L+\gamma W} \tag{11}$$

f. Each element's end forces and moments $\{\hat{f}_{\alpha D+\beta L+\gamma W}\}$ for use in design are then determined according to the following matrix product

$$\{\hat{f}_{\alpha D+\beta L+\gamma W}\} = [\Gamma_{final}]\,[\Gamma_{mid}]^T \left\{[\hat{k}_e][\Gamma_{mid}]\{\delta_{\alpha D+\beta L+\gamma W}\}\right\} \tag{12}$$

in which $[\Gamma_{mid}]$ is as defined above, and $[\Gamma_{final}]$ is the element's global-to-local coordinate transformation matrix based on the system's updated geometry per previous step (e).

The above SIPC procedure, steps (a)-(f), would be repeated for each of the factored load combinations being investigated. It is extremely efficient because only one system of equations (for the corrector step) needs to be assembled and solved for each factored load combination. In contrast, a rigorous iterative-incremental solution scheme could require a computational effort that could be orders of magnitude higher, thereby limiting its usefulness in routine design. That computational cost, however, may come with the benefit of improved accuracy. For this reason, the remainder of this paper focuses on assessing the accuracy of the proposed single-increment predictor-corrector (SIPC) solution scheme.

3. Benchmark Frames

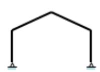
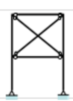
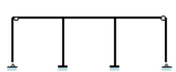
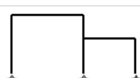
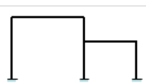
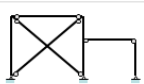
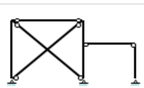
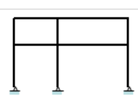
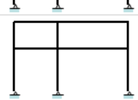
Twenty-two steel benchmark frames were used to assess the accuracy of the SIPC solution scheme. Each of the planar frames (Table 2) has been presented in the literature previously, and as a collection, they represent a variety of practical geometries with a range of sensitivities to second-order effects. All member cross sections are oriented to bend about their major axis (with the exception of two frames), and the structures are all assumed to be fully braced out-of-plane. For each frame, the member sizes, material properties, support conditions, and loading ratios have been duplicated from the original cited work. In this study, however, adjustments from the original load magnitudes have been made to assure that each structure is supporting sufficient load such that its strength is close to its design capacity. This was done to validate the proposed approximate method rigorously, and to ensure that the assessment of its accuracy involved conditions that accentuate second-order effects. In addition, all frames represent realistic designs that satisfy service-load criteria. Detailed information, including member sizes and loading, is available for all benchmark frames by Ziemian and Ziemian (2021b).

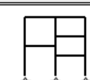
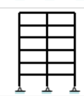
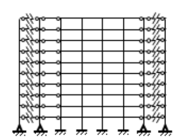
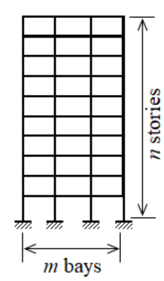
Each frame was modeled and analyzed within MASTAN2 (Ziemian and McGuire 2014). The analysis routines account for second-order effects using an updated Lagrangian formulation and geometric stiffness matrices (McGuire et al. 2000). All members were represented as four planar 6-dof line elements, which has been shown to be sufficient in frames dominated by sidesway behavior and P-Δ effects (Griffis and White 2013).

MASTAN2 was used to perform four different elastic analyses on each of the twenty-two benchmark frames studied.

1. An elastic linear (eigenvalue) buckling analysis (LBA) using the frame's perfect geometry, to compute the critical load factor α_{cr} corresponding to the lowest sway buckling mode of the structure.
2. A first-order linear elastic analysis (LA) performed on the imperfect (out-of-plumb) geometry. The imperfect geometry was used here in an effort to focus comparative studies on the impact of second-order effects, without being confounded by the effect of the initial imperfection.
3. A rigorous second-order elastic analysis with imperfections (GNIA) using an incremental-iterative work control (WC) solution scheme (Yang and Kuo 1994) with a very small step size of 0.001. The WC solution is considered very accurate for the mesh discretization (four 6-dof line elements per member) and the small step size (0.001) used, and it served as the 'expected' or 'exact' elastic solution for comparisons when assessing the accuracy of the SIPC method, i.e. computing percent errors
4. An approximate second-order elastic analysis with imperfections (GNIA) using the proposed single-increment predictor-corrector (SIPC) scheme.

Table 1: Overview of the benchmark frames

Frame No.	Description	Geometry	Load Combination
1	1 story, 1 bay [Ziemian 1990]		1.2D + 1.6L_r + 0.5W
2	Gable; 1 story, 1 bay [Schimizze 2001]		1.2D + 0.5L_r + 1.0W
3	2 stories, 1 bay [Deierlein et al. 2002]		1.2D + 1.0L + 1.0W
4	1 story, 1 bay (10 lean-on) [Maleck 2001]		1.2D+1.6Lr
5	1 story, 2 bays [Martinez-Garcia et al. 2006]		1.2D + 1.6L_r + 0.5W
6	1 story, 2 bays [Martinez-Garcia et al. 2006]		1.2D + 1.6L_r + 0.5W
7	1 story, 1 bay braced, with lean-on [Martinez-Garcia et al. 2006]		1.2D + 1.6L_r + 0.5W
8	1 story, 1 bay braced, with lean-on (minor axis) [Martinez-Garcia et al. 2006]		1.2D + 1.6L_r + 0.5W
9	2 stories, 2 bays [Iffland et al. 1982; Ziemian et al. 1992]		1.2D + 1.6L + 0.5L_r
10	2 stories, 2 bays (minor axis) [Ziemian & Miller 1997]		1.2D + 1.6L + 0.5L_r

Frame No.	Description	Geometry	Load Combination
11	2 stories, 1 bay & 3 stories 1 bay [Schimizze 2001]		1.2D + 1.0L + 0.5L_r +1.0W
12	6 stories, 2 bays [Vogel 1985]		1.2D + 1.0L + 0.5L_r +1.0W
13-16	10 stories, 3 bays; 0, 4, 8, 12 lean-ons [Lu et al. 1977; Statler et al. 2011]		1.2D + 1.0L + 0.5L_r +1.0W
17	10 stories, 5 bays [Lu et al. 1977]		1.2D + 1.0L + 0.5L_r +1.0W
18	20 stories, 1 bay [Lu et al. 1977]		1.2D + 1.0L + 0.5L_r +1.0W
19	26 stories, 3 bays [Lu et al. 1977]		1.2D + 1.0L + 0.5L_r +1.0W
20	30 stories, 2 bays [Lu et al. 1977]		1.2D + 1.0L + 0.5L_r +1.0W
21	30 stories, 2 bays [Lu et al. 1977]		1.2D + 1.0L + 0.5L_r +1.0W
22	40 stories, 2 bays [Lu et al. 1977]		1.2D + 1.0L + 0.5L_r +1.0W

3.1 Stability sensitivity indices

Validation of the SIPC solution scheme, which approximates non-linear behavior with a linear function, required a collection of established benchmark frames that have realistic stiffness and strength, and feature a range of sensitivity to geometric nonlinear effects. As provided in Table 2, three indices were used to indicate the degree to which each benchmark system was stability sensitive.

The first of these indices is the elastic critical buckling load ratio of the frame, α_{cr}, which comes from the LBA and represents the lowest multiplier against the applied load that would result in an elastic instability of the frame in a global sway mode (Walport et al. 2019). The significance of second-order effects may also be expressed as an amplification factor that was originally proposed by Merchant (1954) and computed from α_{cr} as

$$AF_{\alpha_{cr}} = \frac{1}{1-{}^{1}/_{\alpha_{cr}}} \tag{13}$$

For the frames investigated in this study, the α_{cr} values ranged from 1.19 to 7.86, corresponding to range of amplification factors of $1.15 \leq AF_{\alpha_{cr}} \leq 6.22$.

Table 2: Indicators of stability sensitivity for the benchmark frames

Frame	α_{cr}	$AF_{\alpha_{cr}}$	Maximum 2nd- to 1st-order ratios $\frac{\delta^{WC}}{\delta^{LA}}$	Maximum 2nd- to 1st-order ratios $\frac{M^{WC}}{M^{LA}}$
1	7.86	1.15	1.14	1.15
2	6.02	1.20	1.22	1.08
3	1.39	3.58	3.44	3.23
4	1.19	6.22	6.05	2.76
5	3.41	1.42	1.45	1.13
6	5.73	1.21	1.18	1.08
7	6.35	1.19	1.18	1.15
8	2.36	1.74	1.38	1.32
9	2.62	1.62	1.65	1.09
10	1.31	4.27	5.79	5.73
11	4.34	1.30	1.30	1.24
12	6.97	1.17	1.16	1.17
13	6.14	1.19	1.19	1.21
14	3.69	1.37	1.36	1.41
15	2.67	1.60	1.58	1.66
16	2.27	1.79	1.76	1.88
17	4.88	1.26	1.25	1.28
18	6.85	1.17	1.17	1.13
19	5.68	1.21	1.21	1.23
20	5.85	1.21	1.20	1.22
21	6.65	1.18	1.17	1.19
22	6.39	1.19	1.18	1.21

The second index provided is the maximum ratio of the second-order (WC) to first-order (LA) lateral displacements at any joint in the frame. This index, δ^{WC}/δ^{LA}, ranged from 1.14 to 6.05 for the benchmark frames investigated, and was in excellent agreement with the associated $AF_{\alpha_{cr}}$.

The last of the indices used is the ratio of the maximum second-order (WC) to first-order (LA) design moments for all members in the frame, or $\mathrm{M}^{WC}/\mathrm{M}^{LA}$. The design moment in a member is the moment used to proportion the member, i.e. the largest moment along the member's span - considering both directions of imperfection and lateral loading. For the benchmark frames investigated, $\mathrm{M}^{WC}/\mathrm{M}^{LA}$ ranged from 1.08 to 5.73. This index is not to be confused with the largest ratio of second-order to first-order moments at any location along the member's span, which is considered less significant when designing members – unless it includes the maximum or controlling member moments.

4. Results and Discussion

The results of the work-control (WC) and the SIPC second-order analyses are compared to study the response of the benchmark frames as a means for assessing the performance of the SIPC solution scheme. Comparisons are focused on those results believed to be most relevant to the structural engineer, specifically, lateral joint displacements, maximum member design moments, and interstory drifts.

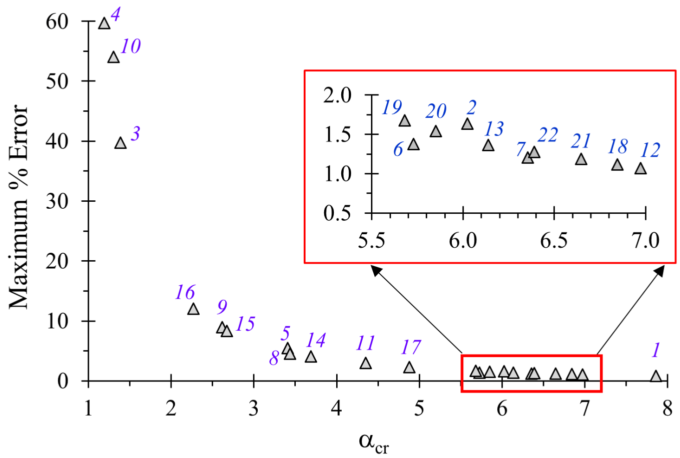
A minor amount of data preparation was done to clean and transform the results prior to processing. This included the filtering of data results from the WC analysis in an effort to avoid the large exaggeration of percent errors that can occur when working with extremely small quantities, which would be considered negligible in the design process. Details of this filtering process are provided in Ziemian and Ziemian (2021a).

4.1 Joint displacements and member design moments

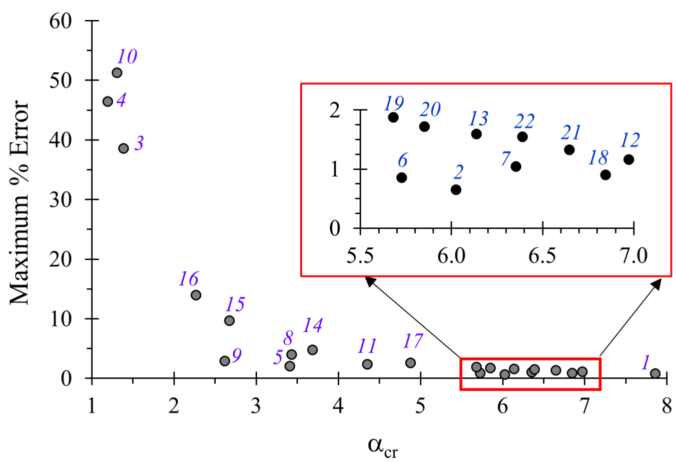
The performance of the proposed SIPC solution scheme in computing lateral joint displacements and member moments was assessed by computing maximum percent relative errors, in which results from the WC solution scheme were taken as "exact". Fig. 2 shows the largest error magnitude for both lateral joint displacements and member design moments within each frame as a function of the frame's critical buckling load ratio α_{cr}.

The maximum (magnitude) error associated with the lateral joint displacements obtained by the proposed SIPC method was below 2% for 11 (or half) of the 22 frames, and below 5.5% for 16 (or nearly three-quarters) of the frames. Similarly, the maximum (magnitude) error associated with the SIPC method's member design moments was below 2% for 11 frames, and below 5% for 17 frames.

As expected, performance of the SIPC solution scheme was strongly dependent on the significance of second-order effects, and the method's accuracy decreased as a frame's response became highly nonlinear. With the SIPC method using a secant stiffness computed at 50% of the system's applied load (Fig. 1), inaccuracies can result when a significant nonlinear response occurs in the equilibrium path between $0.5P_i$ and P_i. The largest errors (over 35%) were associated with Frames 4, 10, and 3, with α_{cr} values of 1.19, 1.31, and 1.39, respectively. Frames with $2 < \alpha_{cr} < 3$ had maximum errors between 3% and 14% when considering both joint displacements and design moments. For the remaining sixteen frames with $3 < \alpha_{cr} < 8$, the majority of maximum error values were below 2%, with the full range between 0.65% and 5.50%.



(a) Maximum (magnitude) percent error associated with lateral joint displacements



(b) Maximum (magnitude) percent error associated with member design moments

Figure 2: Performance of the SIPC solution scheme relative to α_{cr} for each benchmark frame. Labels indicate frame number. (Ziemian and Ziemian 2021a)

4.2 Interstory drift ratios

Given that some specifications, such as the AISC *Specification* (2016), use the ratio of interstory drifts $\Delta^{2ndOrder}/\Delta^{1stOrder}$ as an indicator of the significance of second-order effects, this index was also used to assess the performance of the SIPC solution scheme. Specifically, the maximum percent error for each frame i was computed as

$$\epsilon_i = \max_{s=1:n}(100 \times \left| \frac{\left(\Delta_s^{SIPC}/\Delta_s^{LA}\right) - \left(\Delta_s^{WC}/\Delta_s^{LA}\right)}{\left(\Delta_s^{WC}/\Delta_s^{LA}\right)}) \right| \tag{14}$$

in which Δ_s^{SIPC}, Δ_s^{WC} and Δ_s^{LA} are the interstory drifts for story *s* of an *n*-story system as computed by SIPC, WC, and first-order elastic analyses, respectively.

The comparisons between the interstory drift ratios based on WC and SIPC analyses are further placed into context by providing comparative results with an established story-sidesway amplifier AF_s^{B2} (or B_2). As defined in the AISC *Specification*, in Equation A-8-6 of Appendix 8

$$AF_s^{\mathrm{B2}} = \frac{1}{1 - \frac{P_S}{(P_e)_S}} \tag{15}$$

where P_s is the total vertical load on story *s* (including load from lean-on columns) at LRFD level, and $(P_e)_s$ is the elastic critical buckling strength of story *s*. In this study, $(P_e)_s$ was determined using AISC Specification Equation A-8-7, which is

$$(P_e)_s = R_M \left(\frac{H}{\Delta_H}\right) L \tag{16}$$

where H/Δ_H = story shear stiffness, L = story height, R_M is a stiffness reduction coefficient (LeMessurier 1977), with $R_M = 1 - 0.15\ (P_{mf}/P_s)$ and P_{mf} = vertical load in those columns in story *s* that are part of the moment frames that provides lateral stability (does not include load from any lean-on columns). In this case, maximum percent error for each frame *i* was computed as

$$\epsilon_i = \max_{s=1:n}(100 \times \left| \frac{AF_s^{\mathrm{B2}} - \left(\Delta_s^{WC}/\Delta_s^{LA}\right)}{\left(\Delta_s^{WC}/\Delta_s^{LA}\right)}) \right| \tag{17}$$

in which Δ_s^{WC} and Δ_s^{LA} are the interstory drifts for story *s* as computed by WC and first-order elastic analyses, respectively.

An overview of the interstory drift comparisons is presented in Fig. 3 for all of the benchmark frames in which the story-sway method can be readily applied – this included 16 of the total 22 frames. Frames with irregular geometries and without continuous orthogonal stories were not included because of the complexity and potential errors in computing AISC's B_2 factors. In all analyses, initial sidesway (global) imperfections were included.

The SIPC method was found to be more accurate than AISC's B_2 method for 10 of the 16 frames, with maximum error magnitudes below 5%. All 10 of these frames had critical buckling load ratios of $\alpha_{cr} \geq 3$. The B_2 method was more accurate for those frames with $\alpha_{cr} < 3$, although the associated errors were relatively large, ranging from 3.8 to 26.2%. With significantly smaller error ranges, the SIPC method was more precise than the B_2 method for all but one of the frames. In general, the B_2 method tended to be more conservative than the SIPC method.

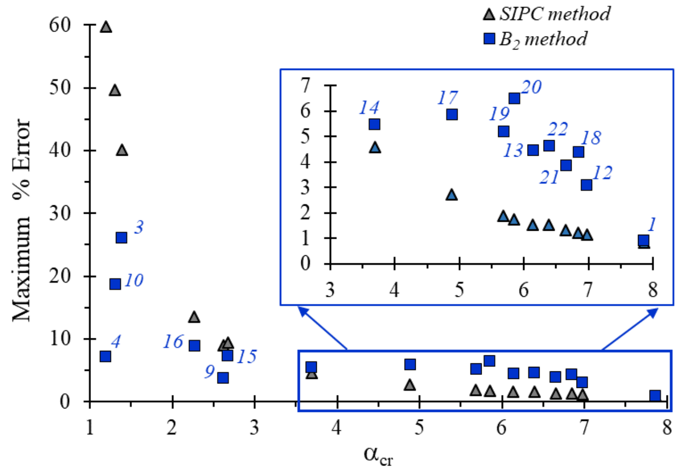


Figure 3: Comparison of the performance of SIPC and B_2 methods in computing interstory drift ratios, relative to α_{cr}, for the 16 applicable frames (Ziemian and Ziemian 2021a).

6. Conclusions

This paper presents a single increment predictor-corrector (SIPC) solution scheme as an approximate geometric nonlinear (second-order elastic) analysis method for use in the routine design of steel and aluminum frames. The proposed method, which is finite element based, is assessed as an alternative to a rigorous and more exact geometrically nonlinear elastic analysis, as well as to an approximate interstory drift analysis that utilizes a well-established amplification factor to account for P-Δ effects. The results of a comprehensive a set of benchmark studies, focused on assessing the validity of a SIPC solution scheme, are provided.

The accuracy of the SIPC method is quite good for the full set of benchmark frames investigated. Specifically, the largest errors associated with lateral joint displacements and member design moments fall within a range of 0.65% to 5.50% for frames with critical buckling load ratios of $\alpha_{cr} \geq 3$. The method's accuracy, however, declines as the value of α_{cr} decreases well below 3. For frames with $\alpha_{cr} \geq 3$, the SIPC method's accuracy and efficiency within the design process make it a very attractive alternative to a more computationally expensive analysis.

In comparing the interstory drift results obtained using the SIPC method with those from AISC's B_2 sidesway amplifier, the SIPC method was found to be more accurate for all frames with $\alpha_{cr} \geq 3$. While the B_2 method was more accurate for those frames with $\alpha_{cr} < 3$, the associated errors were quite large and the use of the amplifiers in design, for these frames, would not be recommended. The precision of the SIPC method was also found to be significantly better than that of the B_2 method, producing significantly smaller error ranges.

The results of this study indicate that the SIPC analysis method provides an acceptable level of accuracy for use in designing typical steel and aluminum structures in which the deformations are not extreme, but are significant enough to require consideration of geometric nonlinear effects. In other words, the results of these benchmark studies in establishing the use of the SIPC method are consistent with the recommendations of EN 1993-1-1 (2005), which indicate the need of a rigorous second-order analysis for frames with $\alpha_{cr} < 3$. Given that the SIPC method can utilize the results of previous analyses used to assess serviceability conditions, and consequently, it requires only one additional linear analysis for each investigated load combination, the method lends itself to an efficient design process when there is a need to evaluate a significantly large number of load combinations and/or rapidly evaluate multiple preliminary designs. Finally, the SIPC method is applicable to a wide range of geometries, well beyond basic tiered structures, and it is not limited to planar frame analyses. This makes it an attractive alternative to a sidesway amplification method, such as B_2, for frames with $\alpha_{cr} \geq 3$.

Future work is planned to explore the use of SIPC in nonlinear time-history analyses. Given that the employed time step is typically quite small, resulting in a small to moderate degree of nonlinearity within a time step, the SIPC method may have great potential.

Finally, it is worth noting that in addition to presenting and assessing the potential of the SIPC analysis method within a steel design process, this study provides extensive results that can be used in future benchmark studies. The MASTAN2 models and all supporting frame details have also been made available online (Ziemian and Ziemian 2021b) to encourage a wide range of additional investigations using these structural systems.

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