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Buckling analysis of castellated steel beams using beam elements

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Abstract
A study is undertaken to determine elastic critical moment of castellated beams. A beam element with warping degree of freedom is developed to study the effects of castellation on lateral tortional buckling. The concept of minimization of the total potential energy is used to derive the stiffness and geometric stiffness matrices. Section properties are calculated for the openings whereas the whole cross section properties are used for the remaining regions. Using this formulation, simply supported and cantilever columns and beam-columns are analyzed various loadings. It is found that the derived beam element generates comparable results with those from the literature. It is well known that if the aspect ratios of the holes are similar the LTB behavior will not be different. However, the present formulation can be used to study LTB of castellations with small and large openings.

1. Introduction

Castellated beams and beams with web openings enhance the strength to weight ratios and are no hindrance to utilities. In spite of all the pros, the higher depth of the cross section for long unsupported, under-construction beam renders it weaker in lateral torsional buckling. The first step in the design of these members is the determination of the elastic critical moment (M_{cr}). Current practice is like that for plain webbed sections, with modified section properties. Properties of the cross section at the center of the holes is assumed throughout the longitudinal direction. This is called the 2T approach. This method is easy to use but underestimates the buckling moment because of reduction in the torsional rigidity of the cross section. For accurate determination, numerical methods are used for which commercial packages are available. Generally, to study for lateral torsional buckling, shell elements are used. But, in this study, a beam element is developed for buckling analysis of members, which easily incorporates the different cross-sectional details of castellated members and gives accurate results.

A review of. literature reveals the trend of study of buckling of castellated members. Researchers (Sonck and Belis 2017) have performed experiments on doubly symmetric hot rolled simply supported, fork boundary conditions, under uniform bending and applied numerical methods for parametric studies using linear buckling analysis (LBA) to calculate M_{cr} and nonlinear geometric and material analysis with imperfections (GMNIA) for the comparison with experimental values,

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during which residual stresses were measured (Sonck, Van Impe and Belis 2014). These exercises were repeated for cellular beams (Sonck and Belis 2015). Similar studies were carried out for axially loaded columns (Sonck and Belis 2016). They have proposed modified torsion constant values, which when used with the 2T approach, result in M_{cr} values comparable with the best fit curve results from their numerical studies.

A finite element program was developed to analyze for deflection and stresses in castellated beams using plane stress elements (Srimani and Das 1978). Analytical formulation for the critical load of a centrally loaded hexagonal castellated column with web shear deformations (Yuan, Kim and Li 2014) was derived.

Based on this literature review, it is identified that either the 2T approach is utilized for M_{cr} calculations, or LBA using commercial software like ABAQUS. Instead of laborious work needed for the earlier mentioned methods, a simple beam element is developed in this paper to calculate M_{cr} (McGuire, Gallagher and Ziemian 2015).

2. Development of finite element

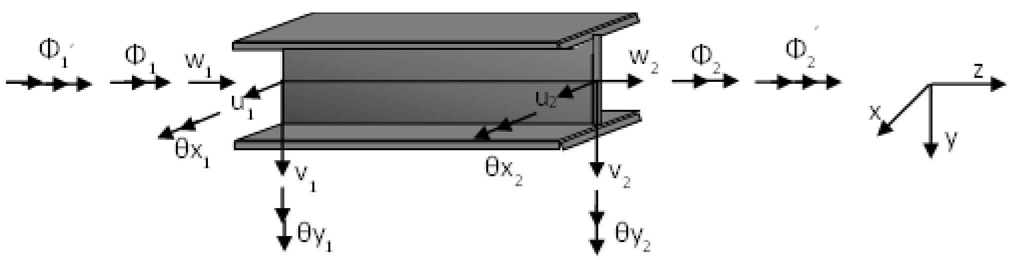

Figure 1: Sign convention for nodal dofs

2.1 FEM formulation

Strain energy and potential energy together form the total potential energy (TPE) expression. Bifurcation buckling occurs when the system reaches neutral equilibrium. Using the theory of minimum potential energy, this condition is satisfied when the second order derivative of TPE with respect to the generalized coordinates is equated to zero as shown in Eq. 1. Pre-buckling deformations are neglected. Hence, only the out-of-plane generalized coordinates are used for the energy equation. Warping degrees of freedom (dofs) are also included in this formulation. Also, unsymmetric cross sections can be solved. The sign convention is shown in the Fig. 1.

$$\frac{1}{2}\int_L \left(\{u'' \quad \phi' \quad \phi''\} \begin{bmatrix} EI_y & 0 & -ES_{\omega y} \\ 0 & GJ & 0 \\ -ES_{\omega y} & 0 & EI_\omega \end{bmatrix} \begin{Bmatrix} u'' \\ \phi' \\ \phi'' \end{Bmatrix} + \{u' \quad u'' \quad \phi\} \begin{bmatrix} F & 0 & 0 \\ 0 & 0 & M_x \\ 0 & M_x & 0 \end{bmatrix} \begin{Bmatrix} u' \\ u'' \\ \phi \end{Bmatrix} \right) dz = 0 \qquad (1)$$

This work employs the standard cubic interpolation functions $[N]$, Eq. 2, for strain displacement relation (Eqs. 3 and 4) for out-of-plane dofs. Here i is the initial node and j is the final node of each element.

$$[N] = \left[1 - \frac{3z^2}{l} + \frac{2z^3}{l} \quad \frac{z}{l} - \frac{2z^2}{l} + \frac{z^3}{l} \quad \frac{3z^2}{l} - \frac{2z^3}{l} \quad \frac{z^3}{l} - \frac{2z^2}{l}\right] \qquad (2)$$

$$\{u'' \quad \phi' \quad \phi''\} = \begin{bmatrix} [N''] & \\ & [N'] \\ & [N''] \end{bmatrix} \{u_i \quad \theta y_i \quad u_j \quad \theta y_j \quad \phi_i \quad \phi_i' \quad \phi_j \quad \phi_j'\} \tag{3}$$

$$\{u' \quad u'' \quad \phi\} = \begin{bmatrix} [N'] & \\ [N''] & \\ & [N] \end{bmatrix} \{u_i \quad \theta y_i \quad u_j \quad \theta y_j \quad \phi_i \quad \phi_i' \quad \phi_j \quad \phi_j'\} \tag{4}$$

Embedding Eqs. 2, 3 and 4 in Eq. 1 and applying numerical integration, the local stiffness matrices are obtained. This approach is in line with Roberts 2004. The local to global transformation is applied which yields the global stiffness matrix (Eq. 5) and geometric stiffness matrix (Eq. 6) corresponding to the rearranged dofs $\{u_i \quad \phi_i \quad \theta y_i \quad \phi_i' \quad u_j \quad \phi_j \quad \theta y_j \quad \phi_j'\}$. This helps to assemble the elements together to form the member. Boundary conditions are applied to the structure to produce partitioned global matrices for restrained and free dofs. The partitioned matrices pertaining to free dof are used to solve for the smallest generalized eigen value of Eq. 8. These eigen values multiplied by the applied load gives M_{cr} values.

$$\begin{bmatrix} \frac{12I_y\mathrm{E}}{l^3} & -\frac{12S_{\omega y}\mathrm{E}}{l^3} & \frac{6I_y\mathrm{E}}{l^2} & -\frac{6S_{\omega y}\mathrm{E}}{l^2} & -\frac{12I_y\mathrm{E}}{l^3} & \frac{12S_{\omega y}\mathrm{E}}{l^3} & \frac{6I_y\mathrm{E}}{l^2} & -\frac{6S_{\omega y}\mathrm{E}}{l^2} \\ & \frac{6GJ}{5l}+\frac{12I_\omega\mathrm{E}}{l^3} & -\frac{6S_{\omega y}\mathrm{E}}{l^2} & \frac{GJ}{10}+\frac{6I_\omega\mathrm{E}}{l^2} & \frac{12S_{\omega y}\mathrm{E}}{l^3} & -\frac{6GJ}{5l}-\frac{12I_\omega\mathrm{E}}{l^3} & -\frac{6S_{\omega y}\mathrm{E}}{l^2} & \frac{GJ}{10}+\frac{6I_\omega\mathrm{E}}{l^2} \\ & & \frac{4I_y\mathrm{E}}{l} & -\frac{4S_{\omega y}\mathrm{E}}{l} & -\frac{6I_y\mathrm{E}}{l^2} & \frac{6S_{\omega y}\mathrm{E}}{l^2} & \frac{2I_y\mathrm{E}}{l} & -\frac{2S_{\omega y}\mathrm{E}}{l} \\ & & & \frac{2GJl}{15}+\frac{4I_\omega\mathrm{E}}{l} & \frac{6S_{\omega y}\mathrm{E}}{l^2} & -\frac{GJ}{10}-\frac{6I_\omega\mathrm{E}}{l^2} & -\frac{2S_{\omega y}\mathrm{E}}{l} & -\frac{1}{30}GJl+\frac{2I_\omega\mathrm{E}}{l} \\ & & & & \frac{12I_y\mathrm{E}}{l^3} & -\frac{12S_{\omega y}\mathrm{E}}{l^3} & -\frac{6I_y\mathrm{E}}{l^2} & \frac{6S_{\omega y}\mathrm{E}}{l^2} \\ & & & & & \frac{6GJ}{5l}+\frac{12I_\omega\mathrm{E}}{l^3} & \frac{6S_{\omega y}\mathrm{E}}{l^2} & -\frac{GJ}{10}-\frac{6I_\omega\mathrm{E}}{l^2} \\ & & & & & & \frac{4I_y\mathrm{E}}{l} & -\frac{4S_{\omega y}\mathrm{E}}{l} \\ symmetric & & & & & & & \frac{2GJl}{15}+\frac{4I_\omega\mathrm{E}}{l} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \frac{6F}{5l} & -\frac{6M_x}{5l} & \frac{F}{10} & -\frac{M_x}{10} & -\frac{6F}{5l} & \frac{6M_x}{5l} & \frac{F}{10} & -\frac{M_x}{10} \\ & 0 & -\frac{11M_x}{10} & 0 & \frac{6M_x}{5l} & 0 & -\frac{M_x}{10} & 0 \\ & & \frac{2Fl}{15} & -\frac{2lM_x}{15} & -\frac{F}{10} & \frac{M_x}{10} & -\frac{Fl}{30} & \frac{lM_x}{30} \\ & & & 0 & \frac{M_x}{10} & 0 & \frac{lM_x}{30} & 0 \\ & & & & \frac{6F}{5l} & -\frac{6M_x}{5l} & -\frac{F}{10} & \frac{M_x}{10} \\ & & & & & 0 & \frac{11M_x}{10} & 0 \\ & & & & & & \frac{2Fl}{15} & -\frac{2lM_x}{15} \\ symmetric & & & & & & & 0 \end{bmatrix} \tag{6}$$

$$\frac{1}{2}\{d\}^T([k]+\lambda[g])\{d\} = 0 \tag{7}$$

$$[k]+\lambda[g] = 0 \tag{8}$$

Mathematica (Wolfram Research Inc. 2020) is used for the numerical integration, assembly of global stiffness matrices, eigen value solution and the overall development of the present formulation. Appropriate inputs regarding the nodes, node connectivity, geometric and section properties are fed into the program and the output is easily generated.

2.2 Modeling details

Mesh convergence study for a plain webbed beam shows that four elements are enough to get reasonably good prediction of LTB capacity, but for castellated members, an element is required for every cross-section change. Non uniform meshing helps to capture the section properties better as shown in Fig. 3. For symmetric loading and boundary conditions, only half length of the member is modelled and appropriate boundary conditions are applied at the centre of the members. For a simply supported member with fork supports for torsion, twist and out of plane displacement are restrained at the end and at the middle length, out of plane rotation and warping dofs are equated to zero.

$$\theta y_{midnode} = \phi'_{midnode} = 0 \tag{9}$$

A statistical mean value of 205 GPA for Young's modulus was adopted.

2.3 Geometric and Section Properties

Table 1: Nominal dimensions of castellated members

	b_f (mm)	t_f (mm)	h_w (mm)	t_w (mm)	Length (m)	No. of holes (n)
Nominal dimensions common for each specimen	82	7.4	205.8	5.4	3.15	15
					3.99	19
					6.09	29
					8.19	39

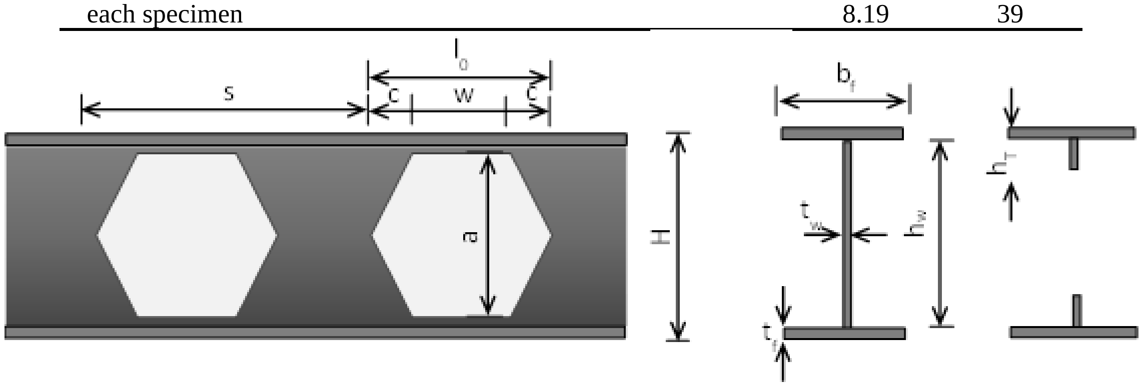

Figure 2: Geometric details

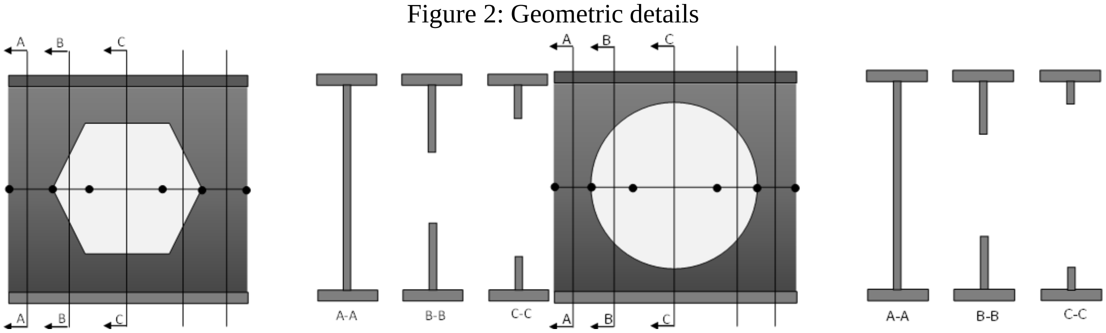

Figure 3: Geometric details for non-uniform mesh

This study involves castellated members derived from IPE160 parent section. The geometric details are shown in Fig. 2. Three shapes of the holes are considered for this study. Section properties are calculated using CUFSM 4 (Schafer and Ádány 2006) and are listed in Table 2. For elements having non uniform cross sections, the properties are calculated at the center of such elements.

Table 2: Section properties of castellated members

Castellation shape	Location	h_T (mm)	A x10^3 (mm^2)	I_y x10^5 (mm^4)	J x10^4 (mm^4)	J x10^4 Ref (mm^4)	I_ω x10^9 (mm^6)
Hexagonal	A-A	-	2.4	6.8	3.3	3.0	7.7
	B-B	80.0	2.0	6.8	3.0	3.0	7.7
	C-C	49.7	1.7	6.8	2.7	3.0	7.7
Circular	A-A	-	2.4	6.8	3.3	2.9	7.7
	B-B	64	1.9	6.8	2.8	2.9	7.7
	C-C	40.3	1.6	6.8	2.6	2.9	7.7
Rectangular	A-A	-	2.4	6.8	3.3	2.9	7.7
	B-B	49.7	1.7	6.8	2.7	2.9	7.7
	C-C	49.7	1.7	6.8	2.7	2.9	7.7

3. Results

3.1 Simply supported beam subjected to uniform moment

The behavior is like the whole beam. As mentioned in the beginning, different shapes of castellation have little influence on the M_{cr} values. The lower bound results are for the 2T approach, followed by rectangular, circular and hexagonal castellation. Though the error with respect to the 2T approach shows up to 6% deviation, comparison with Sonck and Belis 2017 show good match with the current formulation (Table 3).

Table 3: Comparison of results for castellated beams

Castellation shape	L (m)	M_{crFEM} present (kNm)	M_{cr2T} (kNm)	Error[1] (%)	M_{crRef} (kNm)	Error[2] (%)
Hexagonal	3.15	23.4	22.64	3.4	23.4	0.00
	3.99	17.03	16.37	4.0	17.03	0.00
	6.09	10.18	9.71	4.9	10.18	0.00
	8.19	7.31	6.95	5.2	7.31	0.00
Circular	3.15	23.2	22.42	3.5	23.16	0.17
	3.99	16.86	16.18	4.2	16.82	0.24
	6.09	10.06	9.57	5.1	10.04	0.2
	8.19	7.22	6.84	5.6	7.2	0.28
Rectangle	3.15	23.14	22.64	2.2	23.16	−0.09
	3.99	16.81	16.37	2.7	16.82	−0.06
	6.09	10.03	9.71	3.3	10.04	−0.1
	8.19	7.19	6.95	3.5	7.2	−0.14

1. 100 (M_{crFEM}- M_{cr2T})/ M_{cr2T}
2. 100 (M_{crFEM}- M_{crRef})/ M_{crRef}

The modified torsion constant that was arrived at by the best fit curve, proposed by Sonck and Belis 2017 is shown in Eq. 9.

$$J_{avg} = \frac{nl_{0,avg}}{L} J_{2T} + \left(1 - \frac{nl_{0,avg}}{L}\right) J_{full} \tag{9}$$

$$\begin{aligned} l_{0,avg} &= w + 1.5c \\ &= 0.875 l_0 \\ &= l_0 \end{aligned} \tag{10}$$

for hexagonal, circular and rectangular holes respectively. Here, l_0 =140 mm, w =70 mm and c =35 mm.

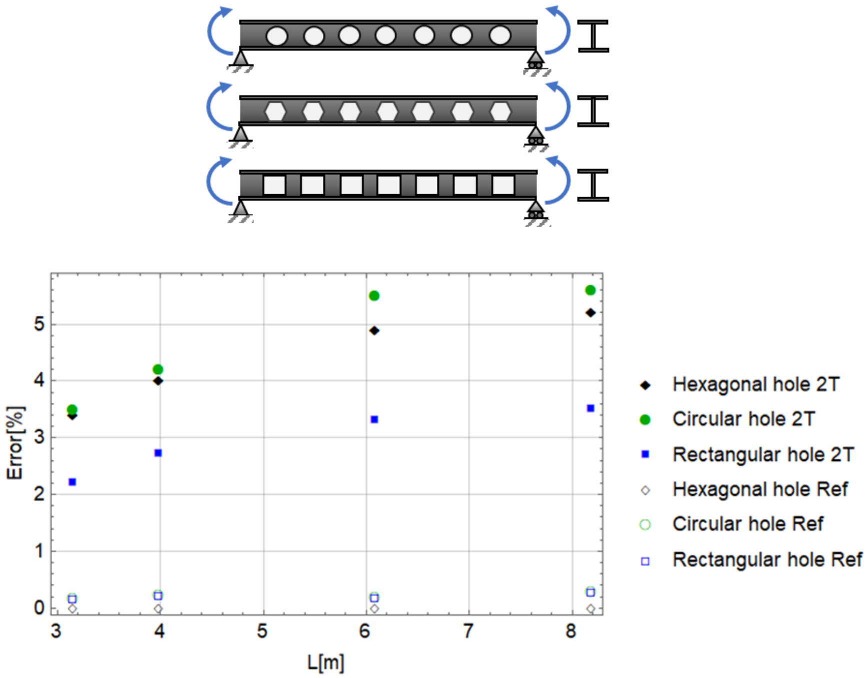

Figure 4: Percentage error of M_{cr} of castellated beam

3.2 Simply supported column subjected to axial load

There is no effect of castellation on a simply supported column subjected to axial load as long as the area of hole is nearly the same.

3.3 Cantilever subjected to axial load

There is no effect of castellation on a simply supported column subjected to axial load as long as the area of hole is nearly the same.

3.4 Simply supported beam-column subjected to axial load applied at different eccentricities from the major axis

For beam column study, only the cellular members are considered. An axial load is applied at eccentricities of 10 cm, 20 cm and 30 cm from the major axis and the degradation of the buckling parameter, as the eccentricity is increased, is visible from the Fig. 5. In another study, the non-dimensional beam slenderness is plotted against the non-dimensional column slenderness. Proportional loading is applied (M/P = 0.1, 0.2, 0.3 etc.) and buckling parameter is found. As the member length increases, its critical axial load as well as the critical moment decreases as shown in Fig. 5. When the eccentricity of axial load about the major axis increases, significant decrease in the buckling load of the column behaviour is observed and vice versa.

4. Conclusions

This work presents a finite element formulation to study the buckling behavior of perfect beams, columns and beam-columns, castellated with different shapes of holes, lengths, loading and boundary conditions. Good agreement is observed with the results from the literature. The intent of this paper is to make a beginning in the use of beam elements to study the complex LTB behavior of beams and frames. It is proposed to extend the formulation into the post buckling regimes.

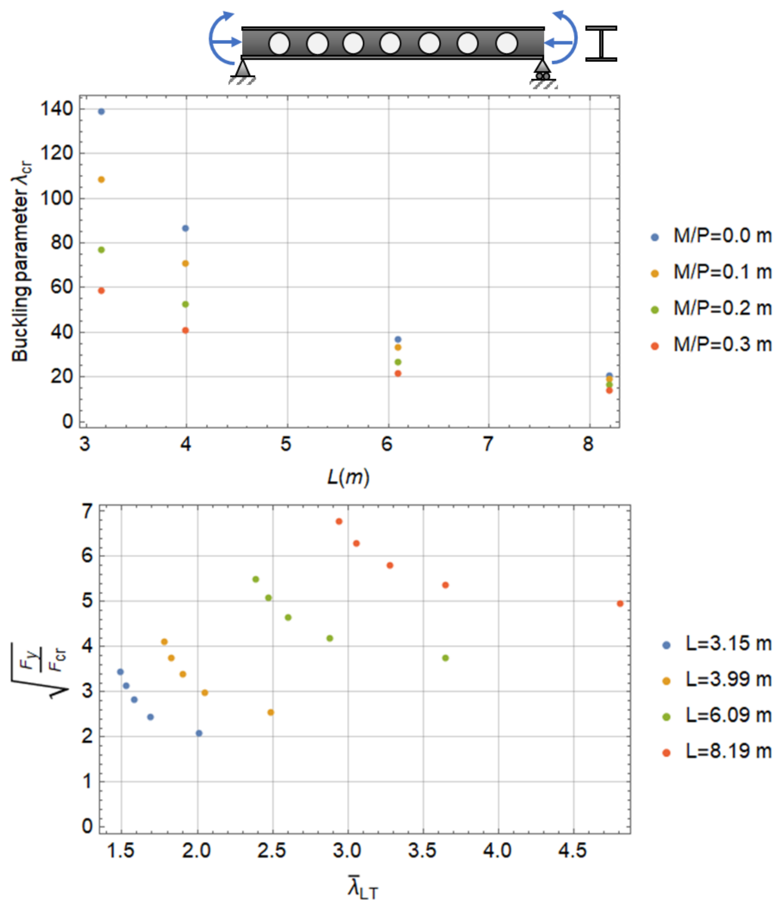

Figure 5: Beam column behavior of cellular members

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