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# Sensitivity of post-buckling behaviour of single layer reticulated shells to loading and member imperfections

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**Abstract**

Research in post buckling of reticulated shells has progressed from solution strategies and formulations to structural optimisation, sensitivity etc. Recent studies have highlighted the sensitivity of the post-buckling behaviour of these structures to topological and member imperfections. The objective of this paper is thereby to determine the influence of member property and loading deviations on the limit point and post buckling behaviour of reticulated domes and the relative importance of these parameters. A deterministic sensitivity analysis is conducted by independently varying each of these parameters. The geometric non-linear analysis to determine stability limit points is based on the corotated - Updated Lagrangian (CR-UL) formulation. The parameter deviations considered in this study are 10% deviation (increase) in member cross-section area (single member and panel) and off-crown loading to levels of 2%, 5% and 10% of the crown load. The influence of the deviations with both elastic and inelastic post-buckling analysis are examined. The results showed that loading imperfections lead to more prominent changes in response to as much as 12% changes in limit loads, while changes to the properties of a single member are less significant. Domes with lesser members are found to be more sensitive to deviations. The influence of deviations is also found to be low when inelastic analysis is adopted. Therefore, when dealing with shallow domes which tend to undergo elastic post-buckling, it would be worthwhile to investigate whether deviations introduced for any purpose have significant effect on the stability.

## 1. Introduction

The stability of reticulated domes is critical due to the shallowness and are generally dealt as a geometrically nonlinear problem. Key geometric parameters that influence the behaviour of such structures are the span-rise ratio, number of members at crown and number of rings. Other effects such as connection semi-rigidity also have a bearing on the post-buckling behavior. Many formulations have been developed and implemented in computer programs for post-buckling analysis of these structures. These procedures include schemes that consider the effect of member yielding and buckling on the critical loads of the structure.

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The effect of member strength and stability are quantified by previous researchers like Papadrakakis (1983), Hill et al (1989), Yang et al (1997), Jayachandran et al (2004), Thai et al (2009) etc. The general theme and objective of these studies were the overall collapse prediction of the domes along with development of improved formulations. The development of robust solution procedures to overcome limit points has also been a part of past research leading to the introduction of procedures such as the Generalised Displacement Control (GDC) and Minimum Residual Displacement Method (MRD). The works of Argyris et al (1982), Crisfield (1981), Bergen and Soreide (1978), Belytschko and Hseih (1973), Chan et al (1988), Yang et al (1997) and Thai et al (2009) may be referred.

Following the establishment of efficient formulations and solution procedures and the availability of modern computation and commercial finite element programs, in recent years the research on these structures has shifted to aspects such as optimization (Ye and Lu, 2020; Lu and Ye 2017), progressive collapse simulations (Tian et al, 2019; Yan et al, 2019; Zhang et al, 2022) etc. Dara (2020) focused on the stability and collapse predictions of single layer reticulated domes under distributed (symmetric and antisymmetric) loading as opposed to classical case of load only at crown.

Research on the influence of deviations in geometric and loading parameters are comparatively limited. A random variable uncertainty analysis by Vazna and Zarrin (2020) and a genetic algorithm based imperfection study by Karimi and Karin (2019) deserve mention. The structural behaviour changes attributed to minor deviations have not been assessed in detail. Among all the possible variables that can have uncertainty (Vazna and Zarrin, 2020), the material parameters, such as yield and ultimate stress were reported to have the least influence, and secondly, the authors wish to point out that as a result of quality in manufacturing, these variables can be sidelined in favour of other variables which can be altered voluntarily. The reference mentioned above also states that loading and member cross-section properties are the first and second most influential variables.

Following the above literatures, the authors of this paper propose the objective as to study the influence of deviations in member cross-sectional area and loading positions on the limit point and post buckling behavior of reticulated domes. The method used in the study is a deterministic analysis, where the sensitivity of the limit point to predetermined changes in above mentioned variables of importance is investigated model-wise.

## 2. Postbuckling analysis using CR-UL formulation

For the post-buckling analysis in this study, the corotated-updated Lagrangian (CR-UL) nonlinear formulation is used which has proven to be efficient in analyzing geometrically nonlinear structures such as single layer reticulated shells. The updated Lagrangian formulation refers all quantities in the finite element equations to the last known configuration as the reference configuration. Detailed explanations regarding the decomposition of deformations into rigid body displacements and stress producing (natural) deformations and their derivations are discussed in detail in Jayachandran et al (2004). A depiction of the CR-UL approach is shown in Fig 1.

The details of the formulation are summarized in Dara et al (2020). The equations employed in the post-buckling analysis are listed from Eqs 1-7. Among them, the first two equations pertain to the geometric nonlinearity, Eqs 3-5 pertain to the material nonlinearity (a bilinear mixed hardening model based on Axelsson and Samuelson, 1979) and Eqs 6-7 account for the buckling of members (Jayachandran, et al, 2004).

$$K_L = \frac{A}{L}(C+\sigma) \tag{1}$$

$$K_G = A^T(E^T K_L E + R_{x2} B)A \tag{2}$$

$$\delta E'^{p}_{xx} = (1-\beta)\delta E'_{xx} \tag{3}$$

$$\delta\sigma_y = MH\delta E'^{p}_{xx} \tag{4}$$

$$\delta\alpha = (1-M)HE'^{p}_{xx} \tag{5}$$

$$F_\Delta = \left[1 - \frac{1}{1+\frac{2}{3}\left(\frac{\delta_c}{L}\right)^2}\right] \tag{6}$$

$$K'_L = \frac{1}{\left[\left(\frac{1}{\frac{A}{L}(C+\sigma)}\right) + \frac{F_\Delta L}{R_{x2}}\right]} \tag{7}$$

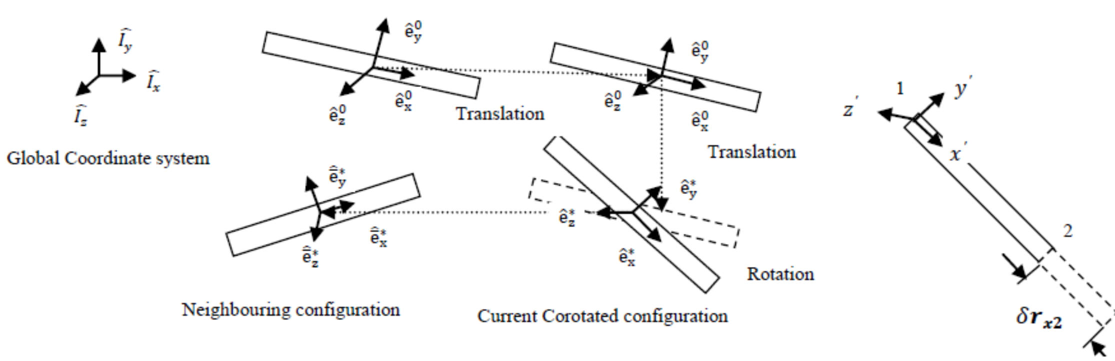



Figure 1: Description of motion in the CR formulation

In the above equations, $C$ is the constitutive matrix, $\sigma$ is the stress tensor and $A$ and $E$ are transformation matrices corresponding to the different configurations in Fig 1. The plastic and total strain increments are $\delta E'^{p}_{xx}$ and $\delta E'_{xx}$, and the portion of elastic in total strain increment is $\beta$. The changes to the subsequent yield stress and stress shift vectors are given by $\delta\sigma_y$ and $\delta\alpha$ in terms of the hardening parameter $M$ and hardening modulus $H$. The reduced member stiffness after buckling $K'_L$ is in terms of the length $L$, member force $R_{x2}$ and midlength deflection $\delta_c$. The minimum residual displacement method by Chan (1988) has proven to be efficient in overcoming limit points such as snap-through typical of reticulated shells and is used in this program as the solution procedure. The program is employed to conduct both elastic and inelastic analysis of reticulated shells. The accuracy of the program has been validated in previously published articles and thus validation is not repeated here.

## 3. Model properties and Parameters considered

A deterministic analysis is used in this study and the basic parameters span-rise ratio, axial stiffness (cross-section area) uniformity and load imperfection are the only ones selected for study. The geometries of the domes are generated for different numbers of members joining at the crown, and number of rings. For all dome models, the cross-sectional area of members are 325 $mm^2$, Elastic Modulus $2 \times 10^5$ $N/mm^2$ and the reference load used is 20 kN.

The geometry of domes used in this study are of the Lamella type. A total of 27 domes are generated by varying the parameters namely, span-rise ratio, number of rings and number of radial members at crown. The number of rings chosen are 2, 3 and 4. The number of radial members at crown chosen are 8, 16 and 24. The span/rise ratios chosen are 5, 7 and 9. The nomenclature adopted is LDXYZ, where LD stands for Lamella Dome, X is the span-rise ratio, Y is the number of members at the crown and, Z is the number of circular rings.

The parametric variations adopted in the study consists of the following for each one of the 27 domes: (i) a 10% increase in cross-sectional area of member at the crown, at a member of the top-most ring and of all members in one panel of the crown region (resembling stiffer members provided around a skylight opening) and (ii) Three cases of off-crown loading, with 2%, 5% and 10% of crown load acting on a node of the first ring, maintaining a total of 100% of the load applied for the crown-only loading case. All the domes are studied by elastic post-buckling analysis first and then inelastic post-buckling analysis to determine the effect of the parameters in both cases.

## 3. Sensitivity of Parameters: Elastic Post-Buckling

### *3.1 Member cross-sectional area*

Elastic post-buckling analysis is first taken up for the 27 domes. The variation with respect to each of the parametric changes are studied one by one, with the first being the effect of increase in crown member cross-sectional area. The member selections for cross-sectional area increase are shown in Fig 2. Typical graphs for the load vs crown displacement are shown for domes LD582, LD 583 and LD584 and LD582, LD5162 and LD5242. In elastic post-buckling analysis, with crown only loading, the span-rise ratio of the top portion of the dome (rather than the overall span-rise ratio) assumes significance as seen from Fig 3(a). In these figures the letter ‘A’ in the legend denotes the series for domes containing crown member with increased cross-sectional area, while ‘R’ denotes that for a member in the topmost ring. For a fixed overall span-rise ratio, the limit load experiences sharp reductions with more number of rings, due to greater shallowness of the top portion. This ‘zonal effect’ is also explained in Dara et al (2020). This relates to a classic case of joint instability where the top ring portion of the dome alone undergoes snap-through which is always true for crown only loading. For a fixed number of rings and span-rise ratio, addition of members to the crown joint enhances the rigidity of the portion which results in viable increase in the limit point, and this is also seen from the figures.

Fig 3(a) shows the load vs crown displacement without any increase of member cross-sectional area while Fig 3(b) and Fig 3(c) show the same for the domes having 10% increased cross-sectional area for a crown member and top-most ring member, respectively. As seen from the figures, magnitude wise, the gain in limit load is not detectable in the figures. Hence, bar charts are provided in Figs 4(a) and 4(b) displaying percentage values of the gain in limit load, for increase in a crown member and a ring member cross-sectional area respectively. The gain in limit load is constant for any given number of crown members while for a given number of rings, the percentage gain decreases with increase in number of crown members. This ranges from 0.5% for 8 crown members to 0.1% for 24 crown members. Though there are changes in deflection due to changes in span/rise ratio, they are of very small quantity that they are negligible.

Greater limit load gain is seen with provision of increased cross-sectional area in a member of the top most ring. The additional restraint offered against outward (circumferential) expansion of the top-most ring by increased area of one of its members tends to lessen the displacement of the crown member connected to this ring member. This results in a raise in the limit point. This is in contrast to the limit load gain obtained by increasing cross-sectional area of a crown member, which is directly due to increased axial stiffness of the crown member itself.

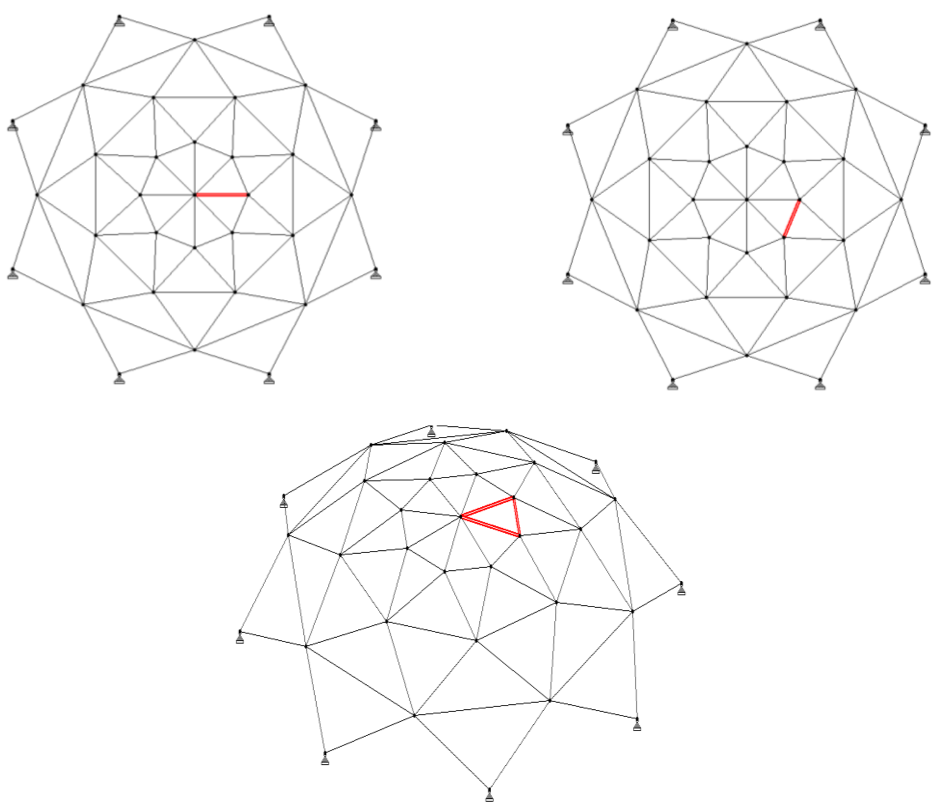


Figure 2: Members selected for cross-section variation: (top left) crown member, (top right) ring member and (bottom) all members in a panel of top-most ring

The effect of increasing the cross-sectional area of any member below the top-most ring is negligible. Specifically, since only the top-most ring participates in snap-through under crown-only loading, increasing the cross-sectional area of any member below this region leads to exactly the same result as the case with no increase in member cross-sectional area. Since these effects are negligible, the related figures are not shown here. Therefore, it can be stated from elastic analysis results, marginal deviations in member cross-sectional area produce little to no effect on the structural behaviour of the reticulated dome.

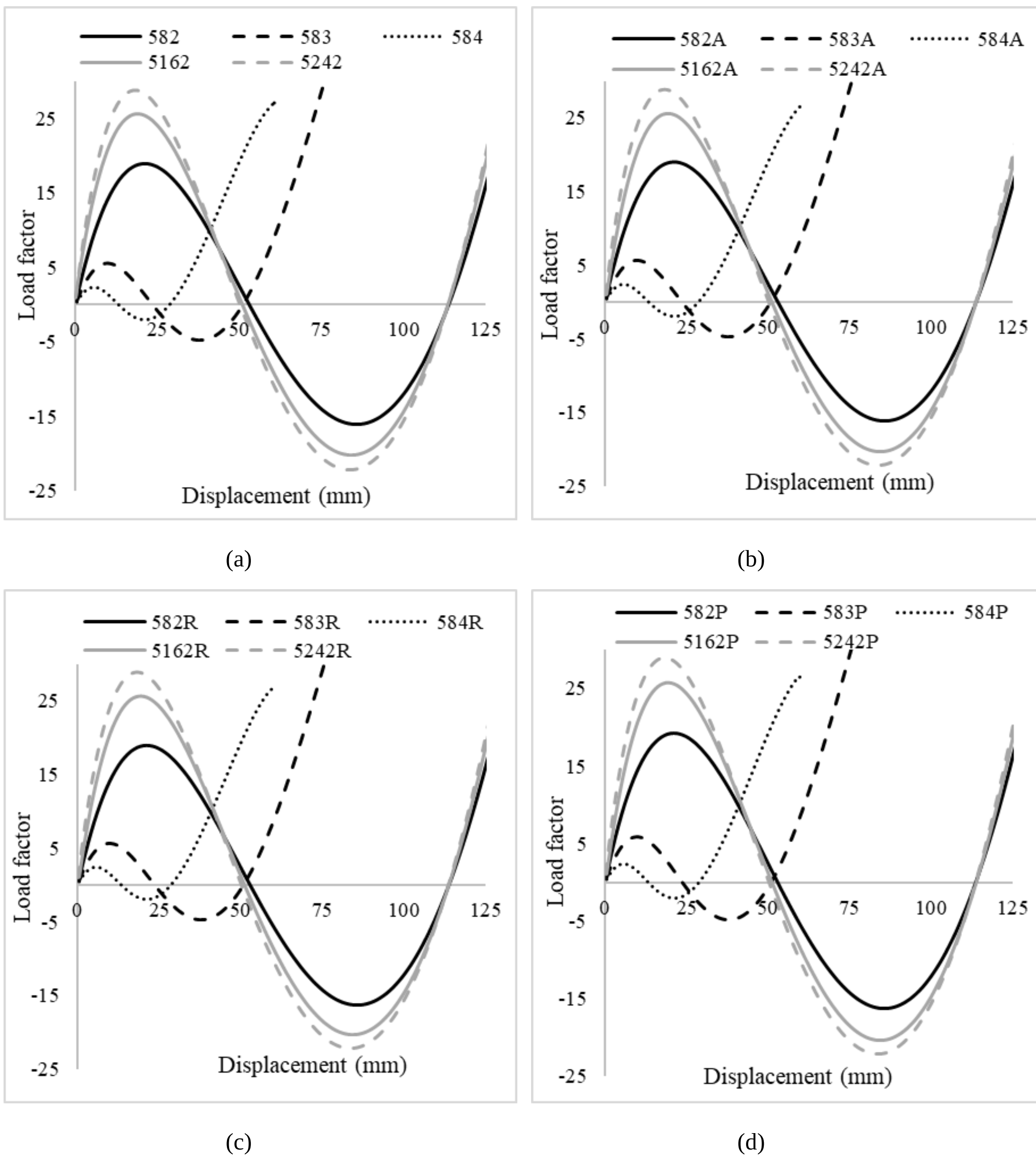


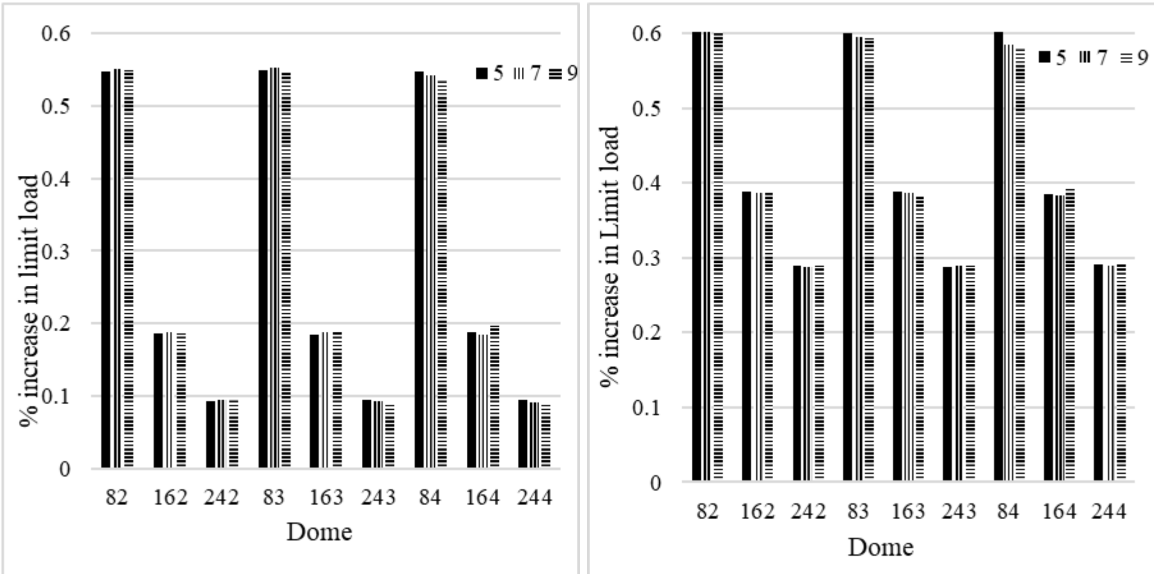

Figure 3: Load deflection of domes of span-rise ratio 5 (a) original (b) 10 % cross-sectional area increase in crown member (c) ring member and (d) 3 members of a topmost ring panel

Observing that the impact of change in cross-sectional area of only one member produces little to no difference in the limit load, increased cross-sectional area was next applied to three members forming a panel in the top-most ring. This arrangement resembles the provision of stiffer members around features such as skylight openings in domes. The arrangement of the three adjacent members forming a triangle like opening at the top is shown in the bottom of Fig 2. In the figure, the members for which imperfections of 10% increase in area are introduced are highlighted by bold line elements. Elastic post buckling analysis is performed on these domes as done for the previous examples. Fig 3(d) shows the related load-crown displacement of the domes with span-rise ratio equal to 5 with varying number of rings and crown members. In these figures the letter ‘P’ in the legend denotes the series of domes containing 3 members of a panel having increased cross-sectional area. Identical observations can be made as that for the series of domes that had one crown member of greater cross-sectional area, except that now, the percentage increase in limit load is much larger. A comparison of the increase in the limit load between domes of series ‘A’, series ‘R’ and series ‘P’ are shown in Fig 5(a) and Fig 5(b), ordered by span-rise ratio 5 and 9 respectively. On comparison, it is seen the percentage increase in limit load due to increase in cross-sectional area either of a single member or group of members is independent of the span-rise ratio of the dome. Another feature to be noted is that, for lesser crown members, this percentage increase is higher than that for domes with more crown members. These percentage increases are also unchanged for differing number of rings. Table 1 summarises the percentage increase of all the above cases.

Table 1: Percentage increase in Limit Loads due to 10% increase in member cross-section area

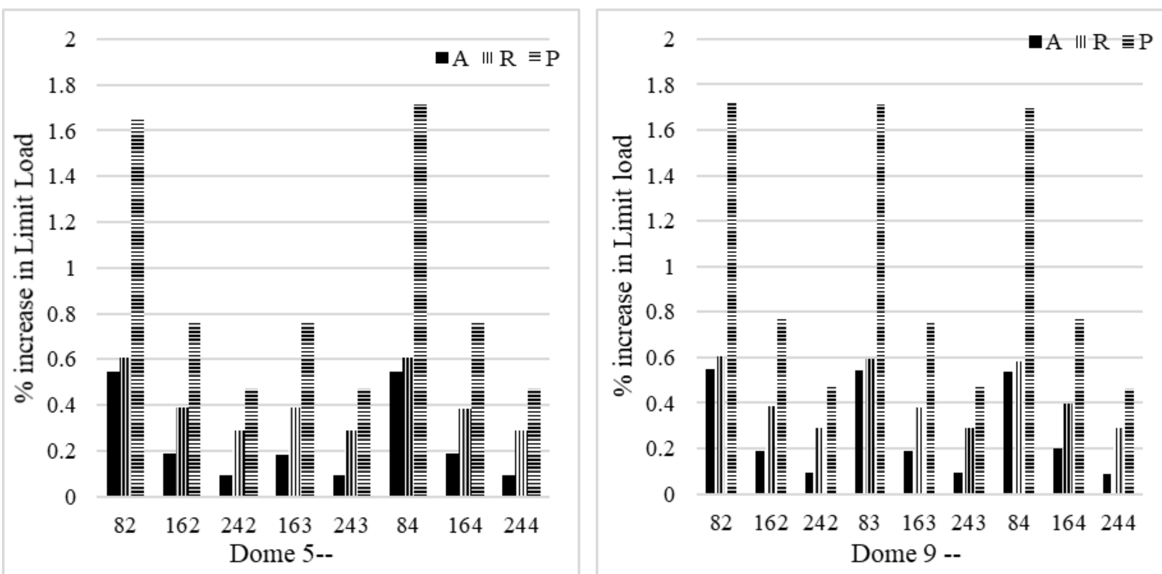
| Span-rise ratio | 5 | | | 7 | | | 9 | | |
|---|---|---|---|---|---|---|---|---|---|
| Dome | Crown | Ring | Panel | Crown | Ring | Panel | Crown | Ring | Panel |
| 82 | 0.55 | 0.61 | 1.65 | 0.55 | 0.60 | 1.73 | 0.55 | 0.60 | 1.73 |
| 83 | | | | 0.55 | 0.59 | 1.71 | 0.55 | 0.59 | 1.71 |
| 84 | 0.55 | 0.61 | 1.72 | 0.54 | 0.59 | 1.70 | 0.54 | 0.58 | 1.70 |
| 162 | 0.19 | 0.39 | 0.76 | 0.19 | 0.39 | 0.76 | 0.19 | 0.39 | 0.77 |
| 163 | 0.18 | 0.39 | 0.76 | 0.19 | 0.39 | 0.76 | 0.19 | 0.38 | 0.76 |
| 164 | 0.19 | 0.38 | 0.76 | 0.18 | 0.38 | 0.76 | 0.20 | 0.40 | 0.77 |
| 242 | 0.09 | 0.29 | 0.48 | 0.09 | 0.29 | 0.48 | 0.09 | 0.29 | 0.48 |
| 243 | 0.09 | 0.29 | 0.48 | 0.09 | 0.29 | 0.47 | 0.09 | 0.29 | 0.47 |
| 244 | 0.09 | 0.29 | 0.47 | 0.09 | 0.29 | 0.47 | 0.09 | 0.29 | 0.47 |

The above percentage based results are insignificant on a practical sense. However they establish that providing increased cross-section area to members not connected to the crown region (where the load acts), leads to no gain in the limit point of the structure. It can be said, under crown only loading, the redundancy provided to the structure by members away from the region plays little role in the instability of the crown joint.




(a) (b)

Figure 4: Effect of 10% increase on cross-section of (a) crown member and (b) top-most ring member




(a) (b)

Figure 5: Comparison between effect of 10% cross-sectional area increases for (a) span-rise ratio 5 and (b) span-rise ratio 9

*3.2 Off-crown loading*

Effect of deviations from perfect crown-only loading is the next aspect. For each of the 27 domes, three types of loading deviations are applied and analysed. They include the following load percentages acting on the crown and one node of the top-most ring, respectively – 98% and 2%, 95% and 5%, and 90% and 10%. The sum of both these loads is maintained equal to the total crown load (20 kN) as applied in the previous analyses. Figure 6 shows this arrangement of crown load deviation.

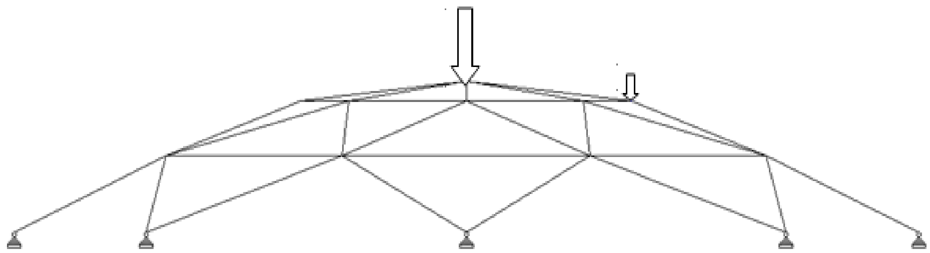


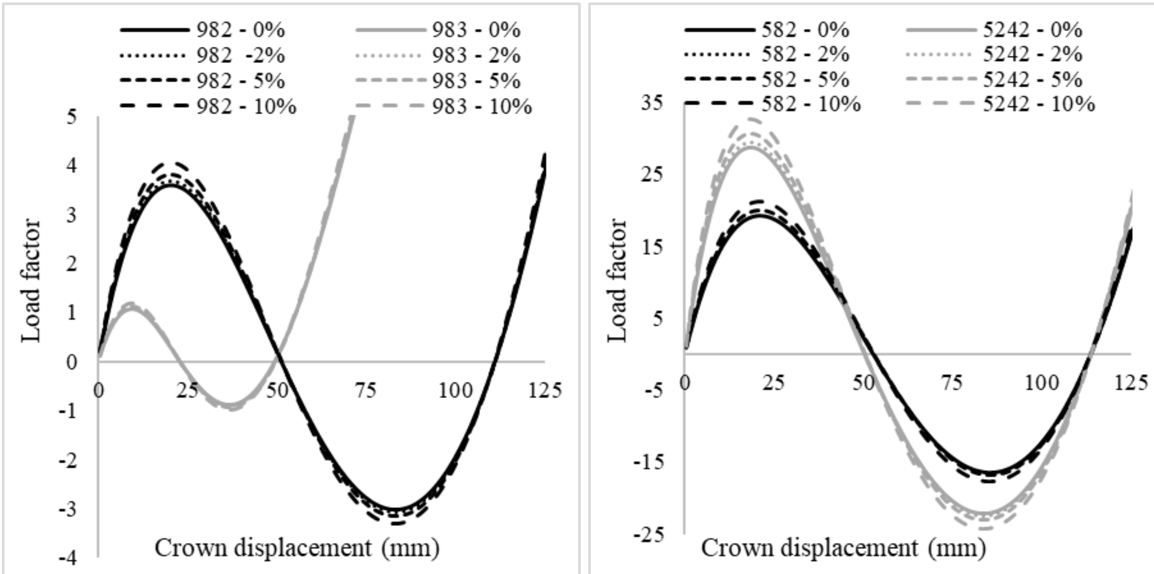
Figure 6: Loading imperfection (Off-crown loading)

The relevant load factor vs crown vertical displacement plots are shown in Fig 7(a) and 7(b) respectively for different number of rings and different number of crown members respectively. In all such cases, the beneficial effect of the off-crown component of the vertical load on the limit point is clearly seen. The greater the off-crown component, the more the increase in the limit load. The mechanism behind this benefit to the limit load is the balancing lever action of the downward load on the ring node, which tends to cause minor uplift on the crown. This mechanism is studied in detail in Dara et al (2020). Fig 8(a) and 8(b) show the increase in limit load for the studied domes. It is noteworthy that the increase in the limit load is very close to the actual percentage of load that has been removed from the crown node i.e., for 98% crown loading the increase is 2.2-2.7%, for 95% crown loading the increase is 5.8-6.6% etc. These percentages for all the domes are listed in Table 2.

Table 1: Percentage increase in Limit Loads due to various percentages of off-crown loading

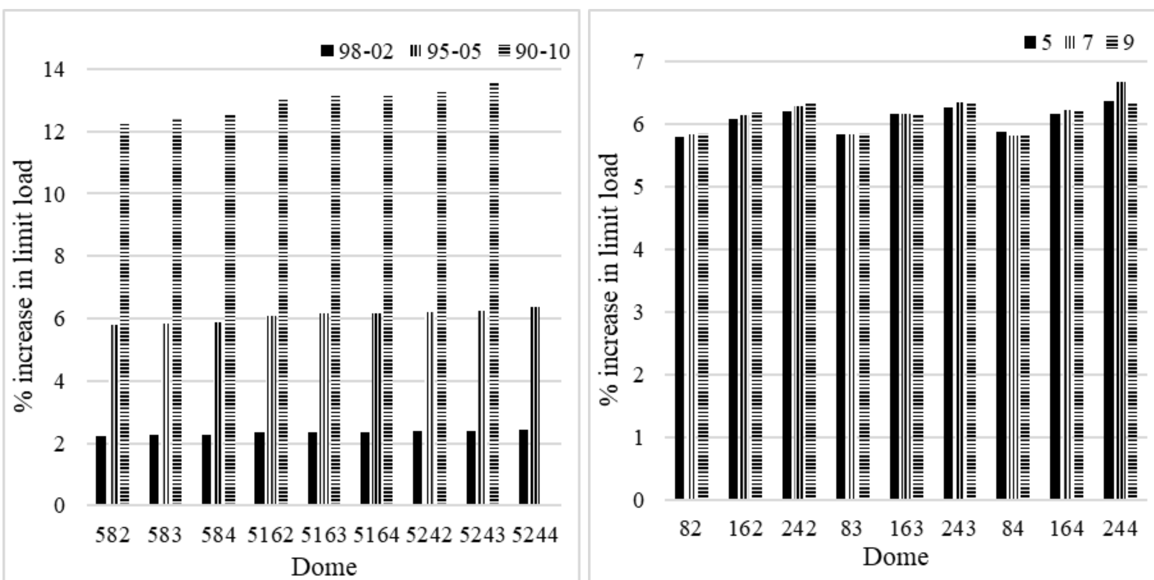
| Span – rise ratio | 5 | | | 7 | | | 9 | | |
|---|---|---|---|---|---|---|---|---|---|
| Dome | 98-02 | 95-05 | 90-10 | 98-02 | 95-05 | 90-10 | 98-02 | 95-05 | 90-10 |
| 82 | 2.23 | 5.78 | 12.30 | 2.25 | 5.83 | 12.41 | 2.25 | 5.84 | 12.45 |
| 162 | 2.25 | 5.83 | 12.41 | 2.26 | 5.84 | 12.43 | 2.26 | 5.85 | 12.46 |
| 242 | 2.26 | 5.87 | 12.52 | 2.24 | 5.81 | 12.35 | 2.25 | 5.83 | |
| 83 | 2.33 | 6.07 | 13.00 | 2.36 | 6.13 | 13.15 | 2.38 | 6.18 | 13.26 |
| 163 | 2.36 | 6.15 | 13.20 | 2.37 | 6.16 | 13.25 | 2.36 | 6.16 | 13.26 |
| 243 | 2.36 | 6.15 | 13.23 | 2.39 | 6.22 | 13.34 | 2.40 | 6.23 | 13.36 |
| 84 | 2.38 | 6.20 | 13.32 | 2.41 | 6.27 | 13.51 | 2.43 | 6.32 | 13.61 |
| 164 | 2.40 | 6.27 | 13.57 | 2.44 | 6.35 | 13.68 | 2.43 | 6.34 | 13.71 |
| 244 | 2.44 | 6.35 | | 2.73 | 6.66 | 14.05 | 2.42 | 6.34 | 13.64 |

This increase is seen to have very little dependence on all other geometric parameters including span-rise ratio, number of crown members and number of rings and can be seen clearly in the figures. The Fig 8(a) shows all domes of span-rise ratio 5 and for all 3 cases of off-crown loading, while Fig 8(b) shows all 27 domes for a 95% crown loading. There is a small gain in the percentage increase observed with respect to the number of crown members. However, this gain is very marginal in comparison to the overall percentage increase caused by the change in crown loading.




(a) (b)

Figure 7: Load vs displacement plot for 2%, 5% and 10% imperfection for different (a) number of rings and (b) number of crown members




(a) (b)

Figure 8: Load vs displacement plot for (a) 2%, 5% and 10% off-crown loading for different span-rise ratio and 5 crown members and (b) all span-rise ratios and 5% off-crown loading

## 4. Sensitivity of Parameters: Inelastic Post-Buckling

### *4.1 Member cross-sectional area*

Inelastic post-buckling analysis was performed on the same set of domes for the same parametric variations as done with elastic post-buckling analysis. The objective is to verify whether geometric and other parametric variations cause changes of comparable magnitude as in the elastic post-buckling analysis. This is worthwhile verifying since inclusion of material strength limits closer represents actual structures. Typical load-crown vertical deflection plots for span-rise ratio 7 domes, obtained from inelastic post-buckling analysis is shown in Fig 9(a). A key difference between the elastic and inelastic post-buckling analysis limit load predictions is the effect of number of rings. As discussed previously, in elastic post-buckling analysis the limit load was lower with greater number of rings due to higher shallowness of the region within the top-most ring, and with limit point directly associated to snap-through within the region. However, with inelastic post-buckling analysis, the snap through is initiated by the strength failure of the members, and hence the rings have a beneficial effect on the limit load.

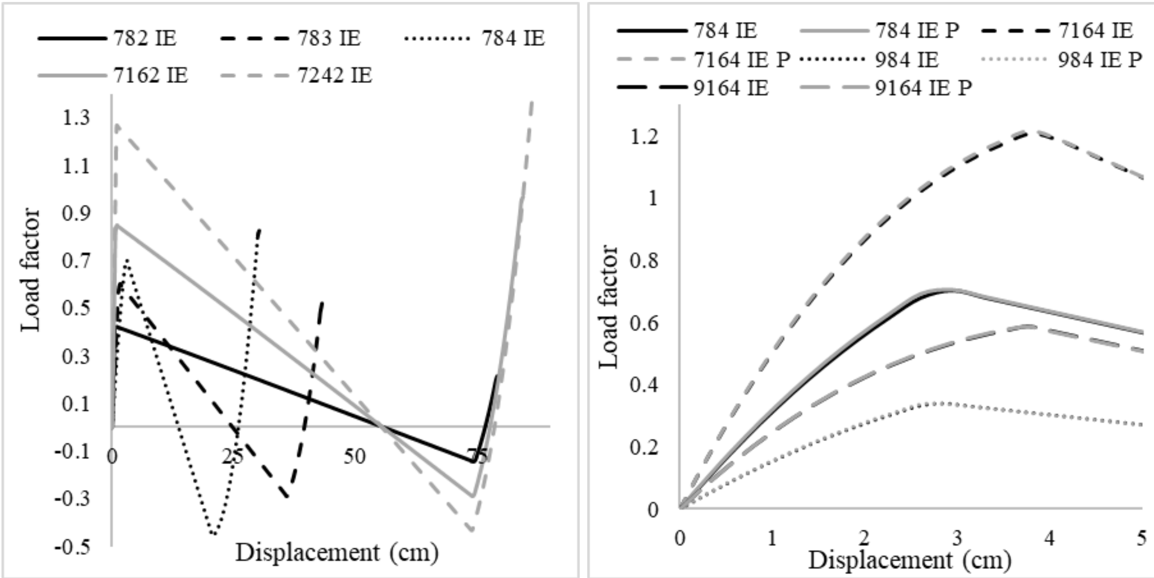
Only the case of cross-section area increase in all members of a panel in the top-most ring is included for inelastic post-buckling analysis, since the case results in the highest rise in limit load. The Fig 9(b) shows magnified and confined part of the load-crown vertical deflection plot near the limit point. It is seen that the percentage increase in limit load is significantly lesser even compared to what was observed with elastic post-buckling analysis. As the limit point in the case of inelastic analysis is decided by member failure, the snap-through limit load which is a global stability limit, will be influenced only if the change in cross-sectional area is enough to cause sufficient force redistribution which can delay the failure of all other members in the top most ring. For example, for dome 782 all crown members failed at a load factor of 0.43 which also becomes the limit load of the dome. It can therefore be stated that while performing inelastic post-buckling analysis, deviations in cross-section area of members can be neglected as it does not influence the limit load of the structure to the extent as while performing elastic post-buckling analysis. The percentage increases for all domes (only case of cross-section area increase in all members of a panel) was found to be in the range of only 0.05-0.1% which is attributable to numerical reasons rather than physical reasons. Therefore, dome-wise comparison figures of the percentage increase in limit load is not shown or tabulated here.

*4.2 Off-crown loading*

The off-crown loading deviation in inelastic analysis was introduced in a similar fashion to that of elastic post-buckling analysis. A benefit to the limit load was once again observed even though the percentage was not as consistent as was observed in elastic post-buckling analysis. The uniformity of the effect of the percentage of off-crown loading for domes of different number or crown members or rings was not uniform as in the previous case. The percentage gain in limit load was generally lower than the former case. Fig 10(a) and 10(b) shows magnified load factor vs crown vertical deflection for span-rise ratio 9 domes with differing number of rings and crown members respectively. Fig 11 represents the percentage wise change in limit load for domes of span-rise ratio 7 and 9. The inconsistency is starkly seen with span-rise ratio 7 where the limit load reduces for greater number of crown members.

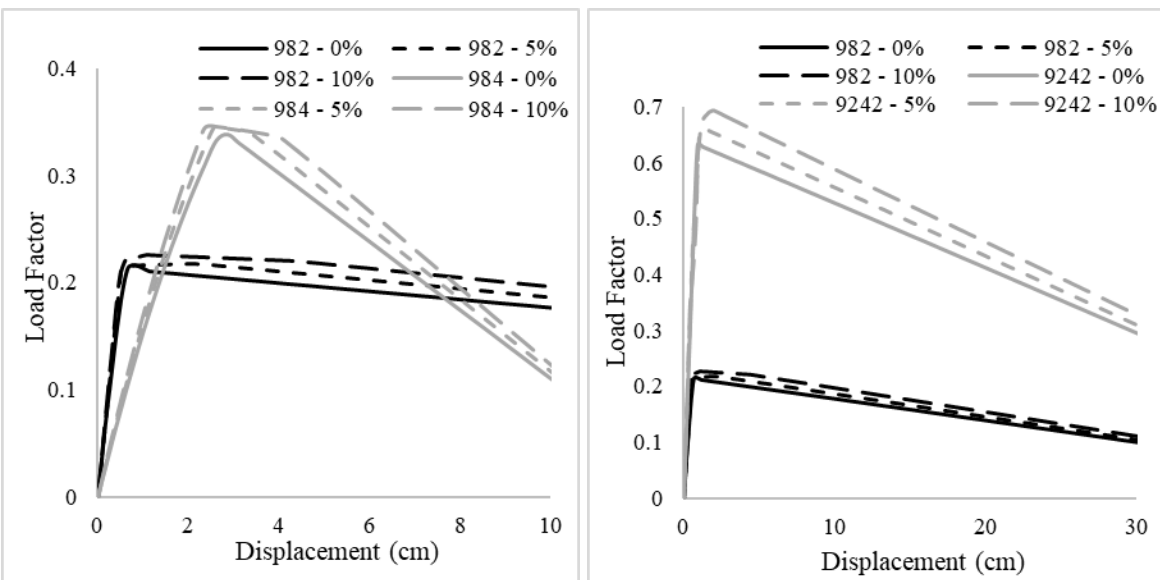
Hence, the limit load is more sensitive to deviation from perfect crown loading as compared to deviation in member cross-sectional area. However, still the sensitivity is smaller in comparison to limit loads obtained from an elastic post-buckling analysis. Overall it can be therefore stated that the sensitivity of member geometry and loading are of greater significance within elastic regime. Even though inelastic post-buckling represents truer scenario due to consideration of member strength failure, in practice large factors of safety and greater shallowness of large span domes would result in global snap-through failure (if it were to occur) only in the elastic regime.

Therefore, it can be said that it is sufficient to study the sensitivity of these two parameters in elastic regime.




(a) (b)

Figure 9 (a) Load vs crown - displacement obtained by inelastic post-buckling analysis span-rise ratio 7 domes (b) effect of 10% cross-sectional area increase of members in a panel of top ring (magnified near limit point)




(a) (b)

Figure 10: Load vs crown - displacement obtained by inelastic post-buckling analysis for off-crown loading cases for (a) differing number of rings and (b) differing number of crown members

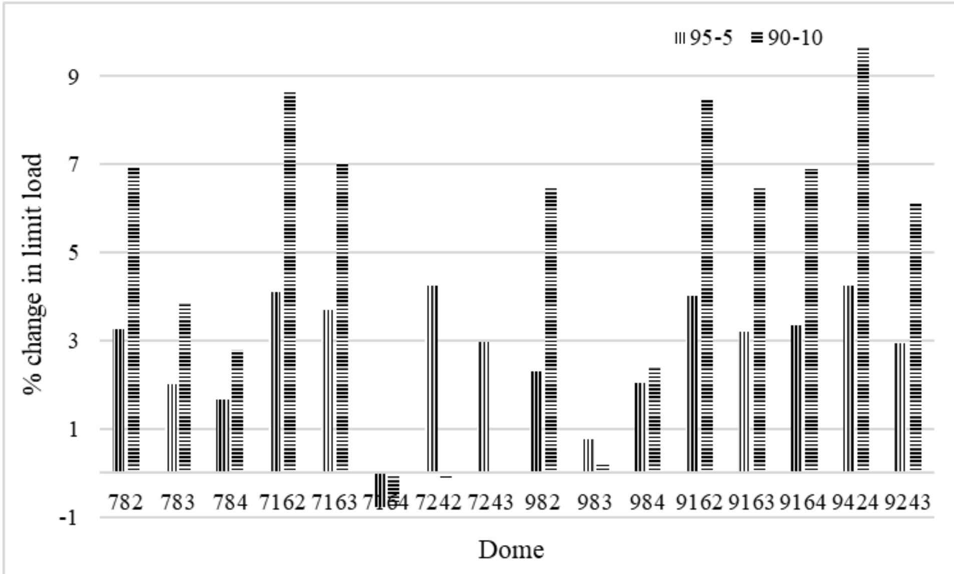



Figure 11: Percentage change in limit load through inelastic post-buckling analysis for off-crown loading cases

*4.3 Sensitivity with respect to load level*

The sensitivity of the parameters was also plotted with respect to different load levels, in addition to only at the limit load. For reasons stated above, this is displayed here only from elastic-postbuckling analysis. It is seen that the vicinity of the zero crossings of the load factor-crown vertical deflection plots see the greatest difference in the load predictions, appearing asymptotic. These carry only numerical interest and from practical point of view this zero crossing point are of low value. Typical graphs are shown in Fig 12(a) and 12(b) for member cross-section area deviation and off-crown loading respectively.

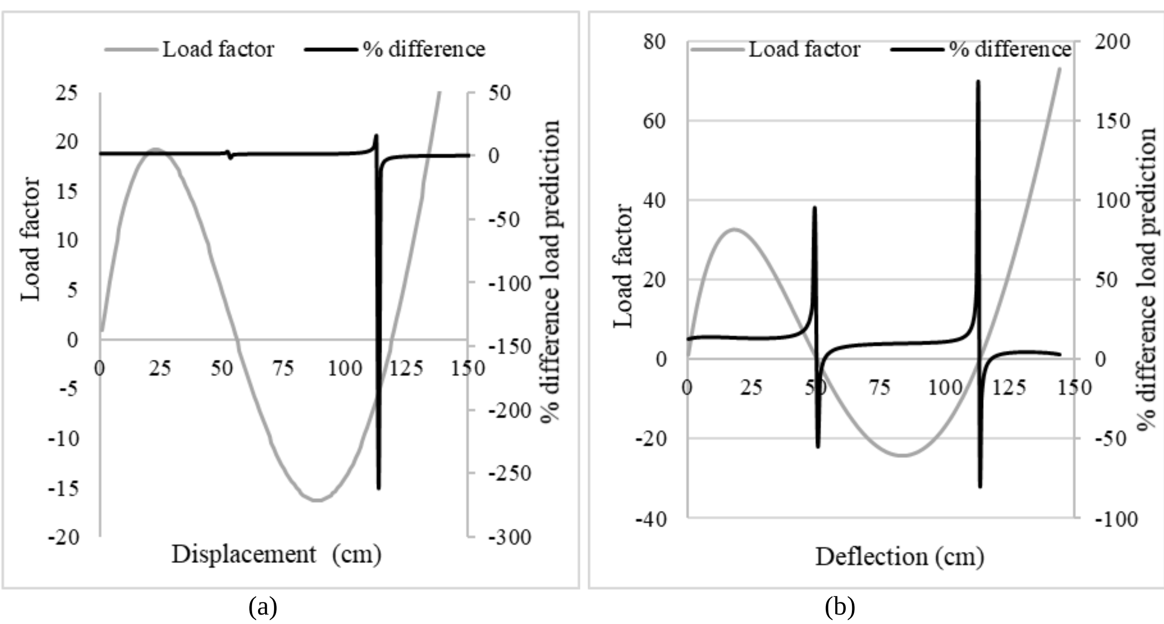



Figure 12: Sensitivity of load prediction with respect to load level (a) Dome 582 cross-sectional area increase in top ring panel (b) Dome 5242 with 10% off -crown loading

## 5. Conclusions

This paper investigated the sensitivity of the limit load of single layer reticulated domes to minor deviations in basic parameters – cross-sectional area of member or group of members and off-crown loading. The investigation was based on both elastic and inelastic post-buckling analysis using a co-rotated updated Lagrangian formulation. The range of parameter values was set from a preliminary analysis. A total of 27 domes, of Lamella geometry, were taken up for analysis individually by introducing variations mentioned above on each of the domes. The variations selected were 10% increase in cross-sectional area of a crown member, a ring member and all members of one panel in the top-most ring (each case analysed separately), and 3 variations of off-crown loading where 2%, 5% and 10% crown load were applied on a node of the top-most ring. Both elastic and inelastic post-buckling analysis was carried out for all the cases. Some of the findings are:

i. Increase in member cross-sectional area of a single crown member has a smaller effect than that for a member in the adjoining ring. This can be attributed to the stiffness against outward deformation of the ring, which delays the snap-through. Increasing cross-sectional areas of all members in a panel produces much more noticeable gain.
ii. In all variations of member cross-sectional area increase, percentage wise, the increase in limit load does not depend on the span-rise ratio or the number of rings. However, the percentage gain depends on the number of crown members and is greater when the number of crown members is lesser.
iii. When there is an off-crown component of loading, there is a gain in the limit load due to a balancing action of the displacement caused by this load to the verticalm displacement of the crown. This percentage increase in load closely follows the percentage of the off-crown load (i.e., percentage of load removed from the crown).
iv. The effect of the 10% increase in member cross-sectional area is much insignificant when inelastic post-buckling analysis is performed, since the limit load is a result of strength failures of other members rather than global stabilty. However, it can be expected that if the cross-sectional area of a member were significantly larger enough to cause force redistribution, there might be noticeable gain in limit load.
v. The effect of off-crown loading, when inelastic post-buckling analysis is performed, is once again to improve the limit load. However, unlike elastic analysis, the percentage change in limit load with respect to the number of crown members and rings is not consistent.
vi. In light of the findings, it can be stated that the effect of minor geometric and loading deviations are significant only when working in the elastic regime. Therefore, when dealing with shallow domes, it would be worthwhile to investigate whether deviations introduced for any purpose, have significant effect.

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