



Buckling and distortion induced fatigue of curved steel plate girders with slender web

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Abstract

The slender webs of modern bridge plate girders are susceptible to buckling in both shear and bending regions. With each passing vehicle cycle, the out-of-plane web deformation due to buckling, fabrication and erection expands – a phenomenon sometimes referred to as “web breathing” – and results in stress concentrations at the intersections of welded elements such as stiffeners. This effect in horizontally curved plate girders is further exacerbated by the non-alignment of the internal forces resulting from major axis (vertical) moment, which push and pull the web out of its plane. While this may not significantly influence the nominal bending strength definition used in bridge design, the resulting fatigue crack potential might affect the service life and future maintenance costs. The objective of the work on which this paper is based is to quantify the stress intensity at risers in curved steel bridge girders that result from the cumulative effects of fabrication, erection, and traffic loadings, and to relate that knowledge to the fatigue limit state. The stress magnitude quantification relies on multi-stage finite element modeling in which the deformation state of the full bridge structure modeled with shell elements is enforced onto high-resolution volume element submodels. Parametric studies using the submodeling technique define the predominant factors and recommendations for preventing and mitigating the fatigue concern.

1. Introduction

Horizontally curved steel bridges provide economical design benefits, span long distances, have shallow depth, and are more aesthetically pleasing than concrete bridges. These structures are the dominant choice for curved highway bridges and comprise approximately thirty percent of all U.S. steel bridges (Zureick et al. 2000). Large scale research effort were initiated in the 1970s and continued into 1990s to investigate behavior of curved steel girder and to develop design guidelines and recommendations (Consortium of University Research Teams 1975; Zureick, Naqib, and Yadlosky 1994; Structural Stability Research Council (SSRC) Task Group 14. 1991). Recently, numerous studies were carried out to further investigate the effect of curvature on the

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stability and strength of slender curved girders for more liberal design considerations (Issa-El-Khoury, Linzell, and Geschwindner 2014; Frankl and Linzell 2017; Broujerdian, Mahyar, and Ghadami 2015; Sanchez and White 2012). Although curved steel girders have been comprehensively researched as a matter of stability and strength analysis, their design behavior for fatigue is still mainly based on straight girder characteristics.

This paper presents ongoing research on the curvature induced fatigue effects on horizontally curved steel girders with slender webs. The preliminary computational results of parametric studies are included.

2. Slender Girder Breathing

When slender plate girders are designed based on post-buckling resistance, large out-of-plane deformations are induced under the in-plane loading. The repeated web buckling deformation, which is known as “web breathing,” leads to fatigue cracks under cyclic loading (Günther and Kuhlmann 2004). It should be noted that inevitable initial imperfections and residual stresses lead to out-of-plane deformations under loads less than theoretical buckling loads (R. Crocetti 2003). Fig. 1 illustrates the breathing mechanism in which buckling deformations lead to high secondary-bending stresses that causes fatigue cracks in the web.

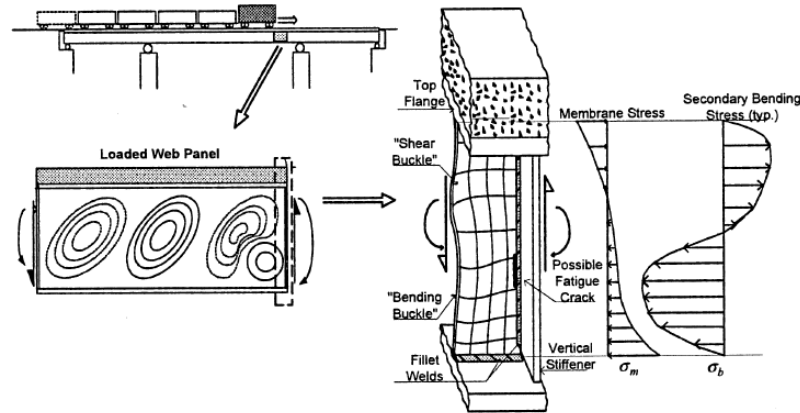


Figure 1: Web breathing effect for a slender composite bridge girder (Roberto Crocetti 2001)

Web breathing fatigue was investigated through analytical and numerical (FEM) studies to quantify the parameters affecting the web boundary stresses (Duchene and Maquoi 1998; Maeda and Okura 1983; Davies and Roberts 1996; Okura, Yen, and Fisher 1993; Remadi, Aribert, and Raoul 1995; Spiegelhalter 2000). It was indicated that the main parameters that affect geometric stresses resulting from web breathing are panel aspect ratio and slenderness ratio, stiffness of boundary members, form, and magnitude of initial out-of-plane deflections (geometric imperfections).

Various experimental tests on slender girders with web breathing indicated that the fatigue cracks could be classified based on the type of loading and place of origin (Spiegelhalter 2000;

Roberto Crocetti 2001; Yen and Mueller 1966; Toprac and Natarajan 1971) as illustrated in Fig. 2. Crack type 1 occurs at the web side of the compression flange mainly because of high secondary bending stress resulted from the in-plane bending moment. The primary source of crack types 2 and 3 initiation is large longitudinal tensile stress. These cracks may propagate from the web to the adjacent members, flange, or transverse stiffener and could eventually cause failure of the girder. Crack type 4 is associated with shear loaded panels due to tension field action. Large out-of-plane deformations in the diagonal direction are anchored at the corners and result in high secondary bending stresses. Crack types 5 and 6 occur when a relatively large load is applied at the transverse stiffener of a panel under combined moment and shear.

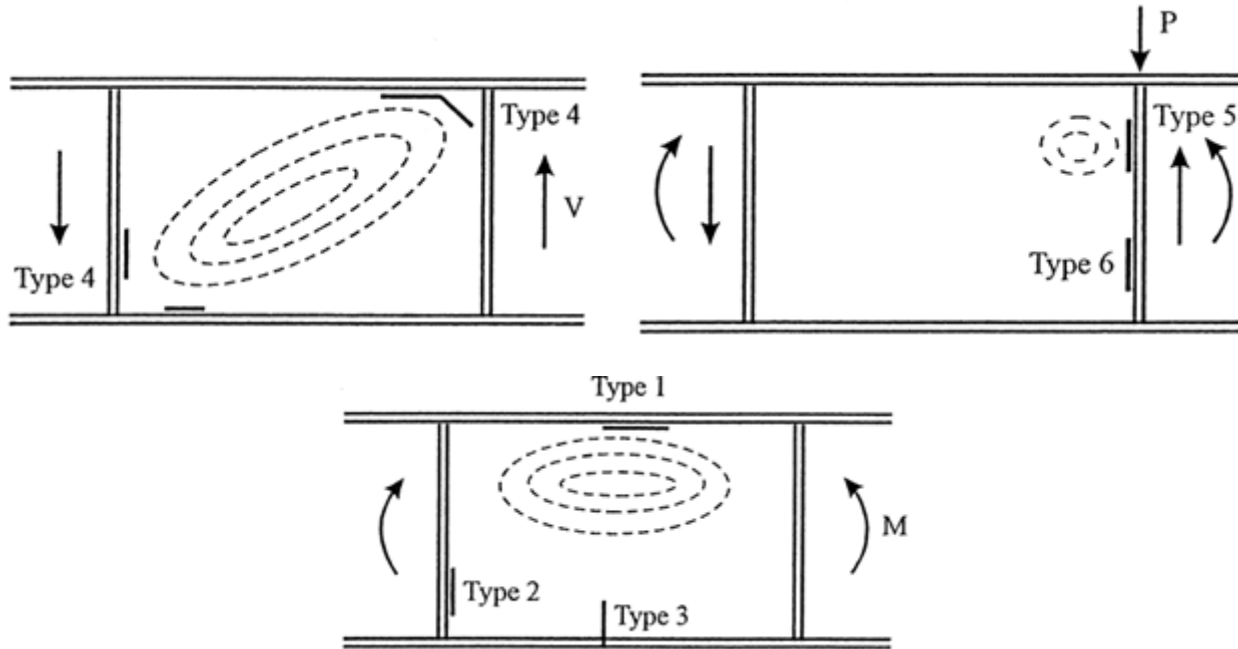


Figure 2: Fatigue cracks locations in slender webs corresponding to loading conditions (Roberts and Davies 2002)

3. Distortion-induced Fatigue

Before the 1980s, it was a common practice not to attach the transverse stiffeners and connection plates to the tension flange to prevent brittle fatigue in the welded flange region. However, another severe fatigue problem appeared in the web-gap region between the short cut transverse stiffener and tension flange. In multi-girder steel bridges, lateral deflection of adjacent girders leads to rotation of the connection plate and distortion of a relatively flexible web gap region. The out-of-plane distortion of the web generates large stress concentrations at the toe of the transverse stiffener and causes fatigue cracking. This phenomenon is called distortion-induced fatigue and accounts for the majority of fatigue cracks in bridges across the United States (John W. Fisher et al. 1990). Fig. 3 illustrates the distortion-induced fatigue mechanism.

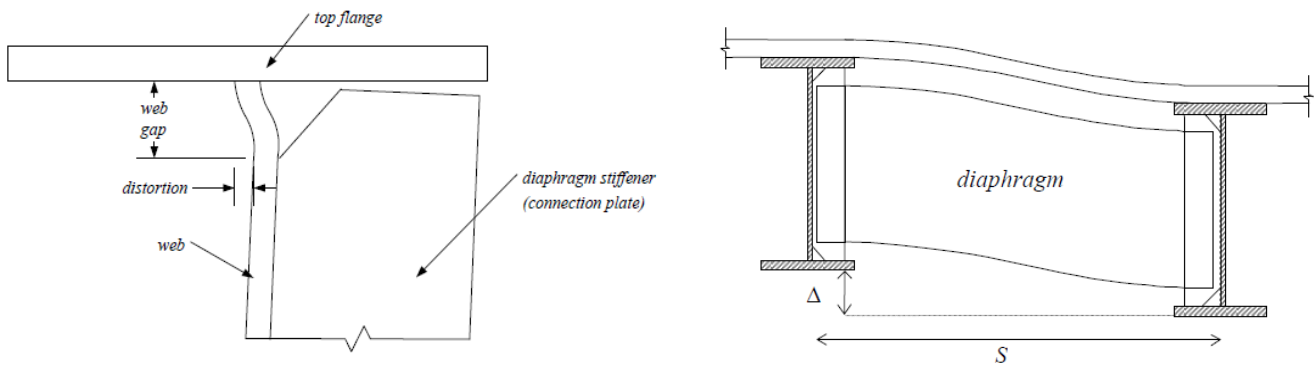


Figure 3: Distortion induced fatigue (Li and Schultz 2005)

There are different views related to critical regions in the bridge where the distortion-induced fatigue is most likely to occur. In a case study, it was found that eight out of nine fatigue cracks occurred in the negative bending moment regions (Khalil et al. 1998). In contrast, another research project suggested that differential deflections and out-of-plane bending moments are maximum in positive bending moment regions (Roddis and Zhao 2001). The major parameters affecting distortion-induced fatigue are lateral bracing system, web gap geometry, bridge geometry in terms of skew angle, span length, slab thickness, and girder spacing (Hassel et al. 2010).

Distortion-induced cracks initially propagate parallel to the major bending axis in the longitudinal direction; hence fatigue failures can be prevented by early retrofitting (J. W. Fisher 1984). Generally, retrofitting methods to prevent distortion fatigue are based on two approaches: (1) stiffening fatigue prone details to minimize distortion, and (2) softening or increasing the flexibility of the system. Diaphragm removal is another method that can be used to mitigate distortion-induced fatigue; however, this does not apply to curved girder bridges due to the primary role of diaphragms to balance the warping and rotations.

4. Curved Girder Fatigue Studies

There are very limited studies specifically related to the fatigue behavior of curved steel girders. The only available sources, as of authors' knowledge, are Daniels and Herbein (1980) at Lehigh University and Nakai et al. (1990) in Japan. Based on experimental tests, a slenderness ratio for load factor design was proposed (Daniels and Herbein 1980):

$$\frac{D}{t_w} = 6.78 \sqrt{\frac{F}{F_y} \left[1 - 4 \left(\frac{d_0}{R} \right) \right]} \leq 192 \quad (1)$$

While no cracks due to geometric stresses at web boundaries were observed during the experiment, fatigue cracks initiated at the location of lateral bracing. It should be noted that limited parameters such as curvature, slenderness ratio, and panel aspect ratio were examined due to experimental setup limitations. Nakai et al. (1990) tested five single curved web

specimens in a curved girder setup system. All five panels had the slenderness and aspect ratio of 144 and 0.7, respectively. Fatigue cracks propagated at the lateral bracing of transverse stiffeners, but no crack was observed at the web boundary.

5. Residual Stress

Residual stresses have a considerable effect on the fatigue life of steel structures (Champoux, Kapp, and Underwood 1988). Generally, surface tensile residual stresses decrease fatigue performance by accelerating crack growth; however, compressive residual stresses may have a positive effect due to reducing the fatigue crack tip stresses (Webster and Ezeilo 2001). The sources of residual stresses in horizontally curved steel girders are welding, flame-cutting, and heat curving. Differential shrinkage during welding and thermal cutting leads to tensile and compressive residual stresses in heated and unheated areas, respectively (Russo et al. 2016). The measured residual stresses for three different grades of steel are shown in the Fig. 4. These stresses are related to residual stresses in the flanges of the welded plate girder parallel with the longitudinal axis of the beam. As can be seen in Fig. 1, the highest magnitude of tensile residual stress occurs at the location of a web-to-flange fillet weld, and is equal to the yield stress. Various models have been developed to determine the residual stress pattern in flanges and webs of the build-up members (Chacón, Serrat, and Real 2012; Barth and White 1998; Clarin 2004; Taras 2010; Kim 2010). Two typical models used for curved girders are ECCS (1976) and Culver and Nasir (Culver 1972), which includes the heat curving residual stresses in addition to flame cutting and welding. Fig. 5 illustrates the ECCS residual stress pattern functions and corresponding equivalent element stresses of an FEA model. In ultimate strength analyses, these stresses are introduced into the FEA model as an initial state. However, residual stresses are not modeled explicitly when the reference S-N approach is applied in fatigue analysis. Residual stresses do not change the stress range, which is the governing parameter in the S-N approach. The effect of residual stress is included in the detail category fatigue resistance (Fisher et al. 1969). In another parametric study (Daniels and Batcheler 1979), the heat curving residual stress effect on fatigue strength of the curved steel girder was investigated. Two mechanisms were studied: mean stress effect and excessive web bowing under compressive residual stresses. It was concluded that “heat curving has no significant effect on the fatigue strength due to either mechanism.” In addition, it was recommended that the entire stress range, including the compressive stress, should be considered if any stress reversal occurs.

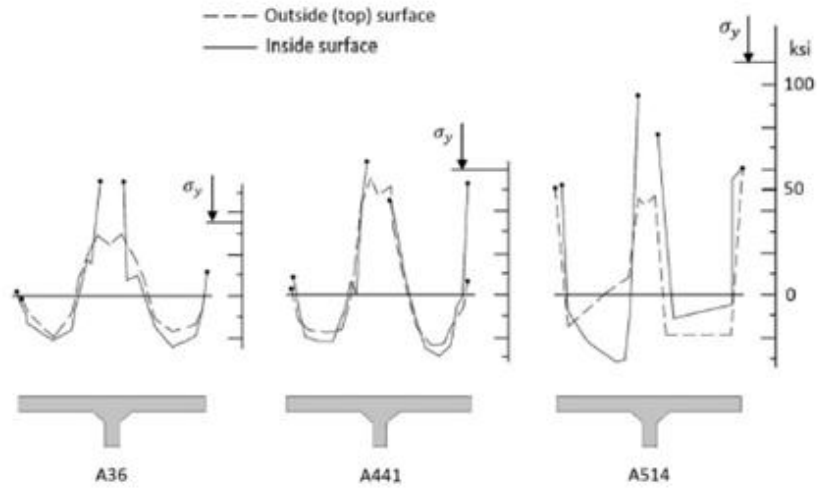


Figure 4: Residual stress distribution in welded plate girder flanges (Keating et al. 1990)

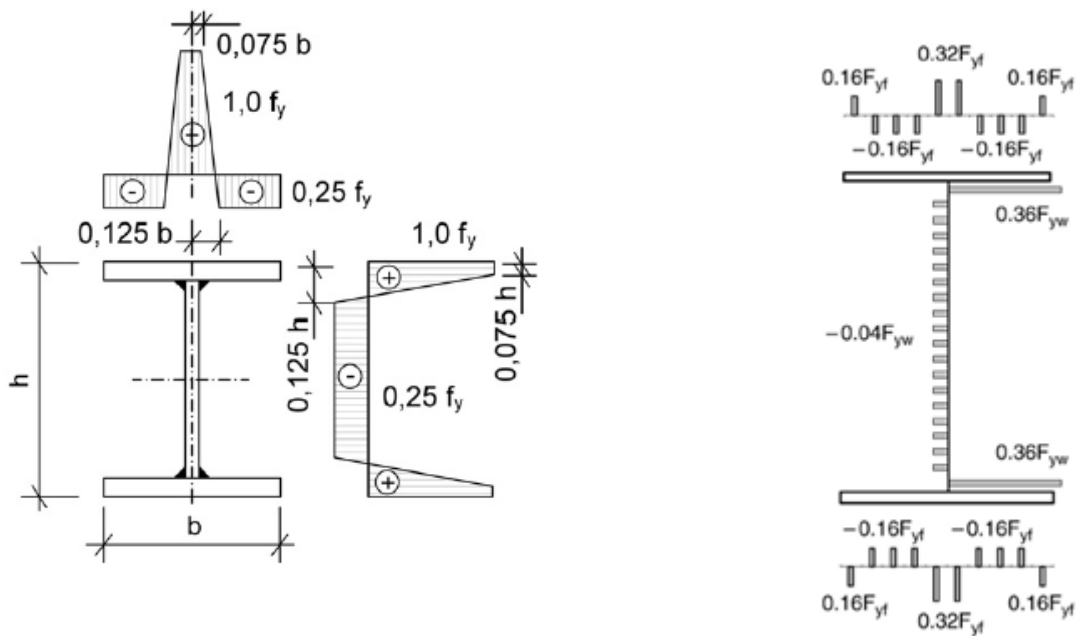


Figure 5: ECCS (1976) Residual Stress model, Left) Stress magnitudes (Pasternak, Launert, and Krausche 2015) Right); equivalent residual stresses averaged over elements of FEA model (Jung and White 2006)

6. Design Specifications

6.1 AASHTO Specification

There are two fatigue categories included in AASHTO (2017): load-induced and distortion-induced fatigue. In the load-induced part, AASHTO limits the web shear force to shear buckling resistance of the web by Eq. 2:

$$V_u \leq V_{cr} \quad (2)$$

Where V_u is shear in the web at the section under consideration due to the unfactored permanent load plus the factored fatigue load (kip) and V_{cr} is shear-buckling resistance. It is assumed that the member sustains infinite fatigue life related to elastic flexing of the web. In the distortion-induced part, the problem is addressed through proper detailing, and no further limit is mentioned.

6.2 Eurocode Specifications

The Eurocode (2006) limits the web slenderness by Eq. 3 to prevent web breathing in road bridges:

$$\frac{b}{t} \leq 30 + 4L \leq 300 \quad (3)$$

Where b is the web height and t is the web thickness, and L is the span length in meters and assumed to be more than 20 meters. It should be noted that both specifications limitations are related to slender web straight girders, and none of them provide guidance specifically to curved steel webs.

7. Finite Element Model Description

The finite element analysis studies were conducted using the commercial FEM software package ABAQUS (2019). Two modeling strategies were utilized to capture the stress ranges corresponding to different fatigue crack types that are developed in curved slender girders. The girder specifications are given in Table 1. In both methods, the AASHTO (2017) fatigue truck was used for the loading configuration. The methods are discussed in the following.

Table 1: Curved steel girder geometrical characteristics

t_w (in)	D (in)	t_f (in)	b_f (in)	d_o (in)	L_b (ft)	R (ft)	Span Length (ft)	Slenderness ratio
0.4	120	1.35	24	120	30	900	240	300

7.1 Modeling technique for capturing the crack conditions related to slender web breathing

The most influential parameter related to the web breathing of slender girders is the initial imperfection of the web. Scaled buckling shape modes of the girder were used as the initial imperfections in this study. First, a linear buckling analysis was conducted to extract mode shapes. The buckling shape mode related to the first eigenvector is shown in Fig. 6. Second, these shape modes were scaled and used as the initial geometry to run a geometrically nonlinear analysis under the same loading. The (ABAQUS 2019) S4R shell elements, having 6 DOF's per node with reduced integration, were used to model the web, transverse stiffeners, flanges, and longitudinal stiffeners.

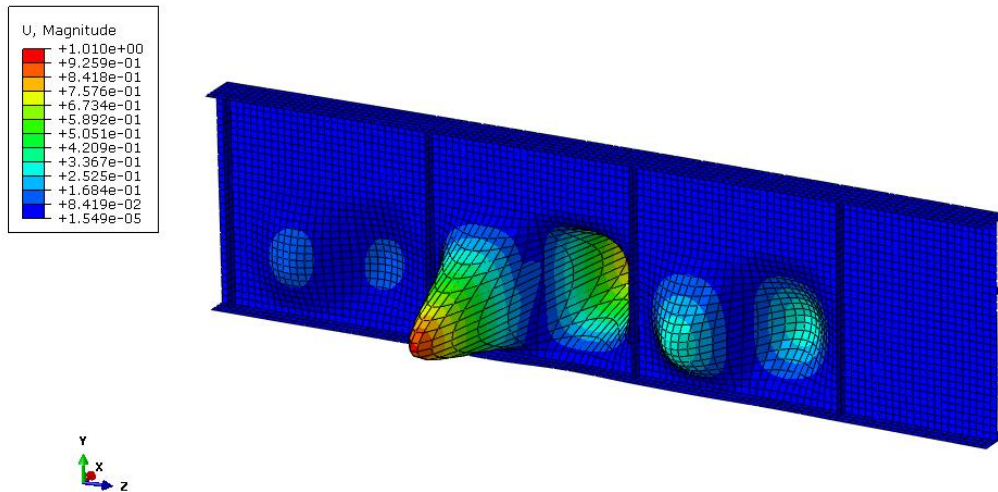


Figure 6: First buckling shape mode as an initial imperfection (magnified deformation)

7.2 Modeling technique for capturing the crack conditions related to lateral bracing of the connection plate

There is a complex stress state at the connection location of the transverse stiffener and flange and web elements. The lateral displacements in the radial direction are restrained at those locations. The bending moment and rotation of the tension flange are the primary causes of the stress complexity. Moreover, the connection details and weld geometry make it more challenging

to get an accurate stress state at the connection location. Hence, a two-level analysis referred to as sub-modeling technique was applied. In the first step, all girder parts are modeled using S4R shell elements in the global model. In the second step, a sub-model of the critical region is regenerated using solid elements, C3D8R, and displacements from the global shell model are applied at the boundary of the sub-model. This method optimizes the computational cost by using shell elements for global analyses and results in accurate stress definitions due to solid element capabilities. Fig. 7 shows the submodel and global model geometry. A very fine mesh of dimensions approximately equal to 0.25 inch for each element was used to precisely calculate the complex stress state due to lateral bracing and detailed weld geometry.

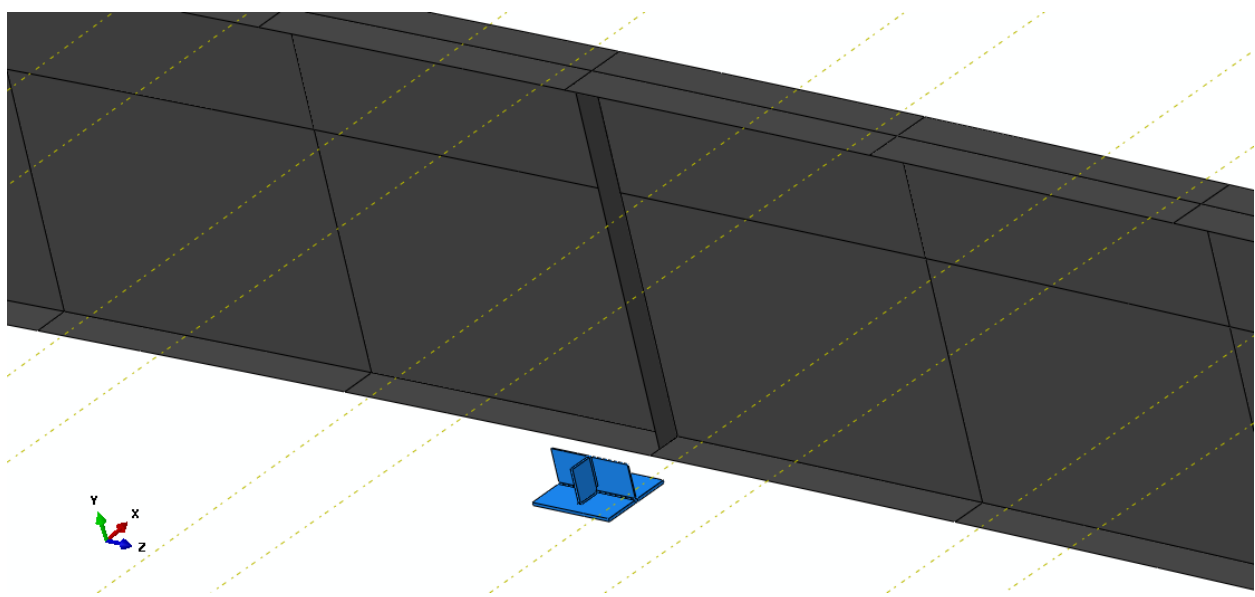


Figure 7: Global model and submodel

8. Results and discussion

8.1 Web breathing

Fig. 8 shows the normal transverse stress, normal stresses to the girder flange, distribution in the girder considering initial geometric imperfections. As expected, these stresses are small close to the top flange due to the stabilizing effect of the longitudinal stiffener. The highest stress range occurs at the bottom flange and is equal to 9.5 ksi. The fatigue resistance of the web-to-flange connection, along the horizontal web boundaries, is equal to 16 ksi (Günther 2002). Hence, no fatigue cracking due to the web breathing effect is expected at the web boundary. It is noteworthy that the maximum stress ranges occurred at the location of maximum initial imperfections, refer to Fig. 6, showing the significant role of geometric imperfections.

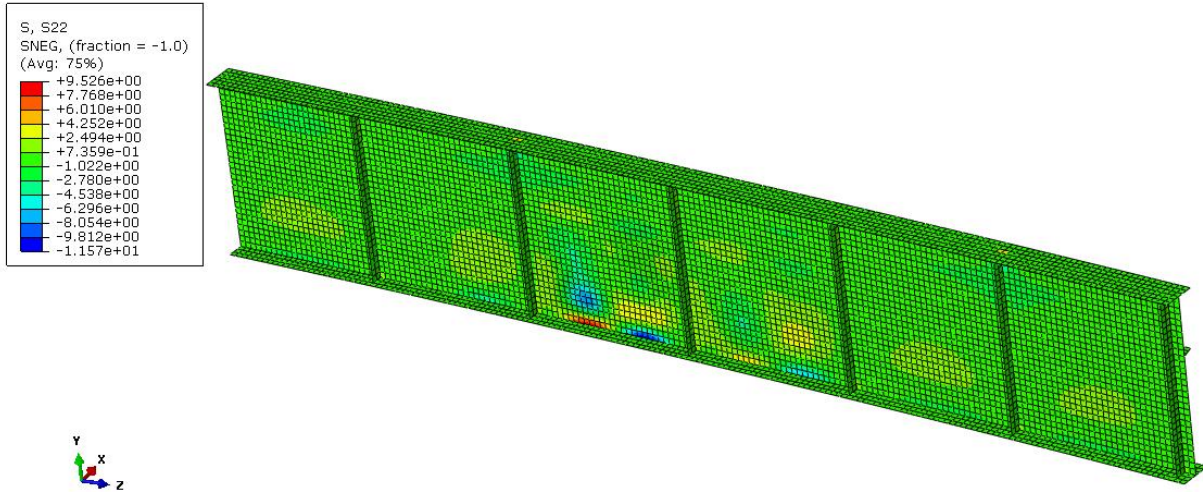


Figure 8: Normal stress distribution in Y direction, normal to the flange, in ksi

8.2 Web-to-flange-to-stiffener connection

Fig. 9 shows the normal stress distribution along the longitudinal axis of the girder resulting from the transformation in the tangential direction of a cylindrical coordinate system. As can be seen, the stress magnitude is high, approximately 46 ksi, at the stiffener-cut location connected to the web. The fatigue resistance of the detail is equal to 12 ksi based on AASHTO (2017), hence it indicates that this region is susceptible to fatigue cracking.

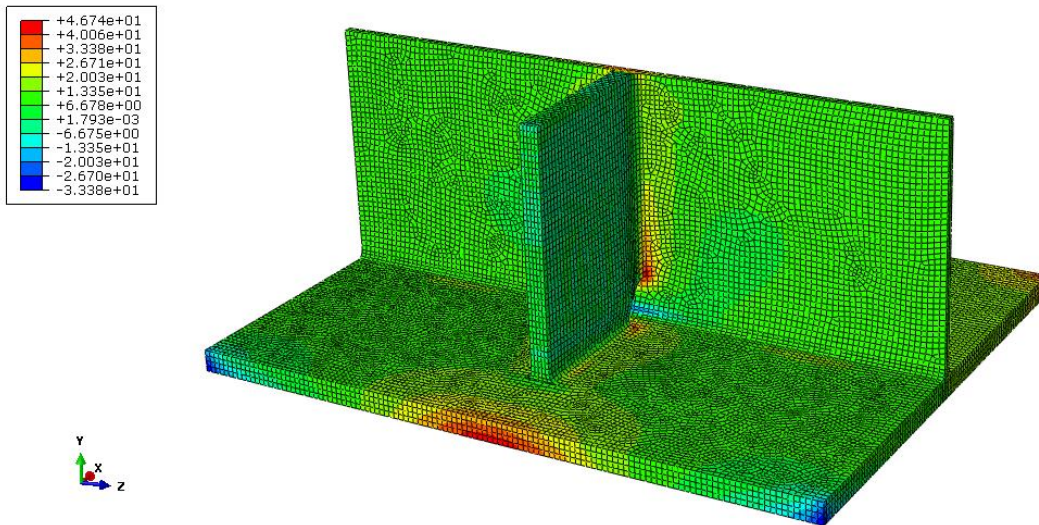


Figure 9: Tangential stress, longitudinal, distribution of submodel in ksi units

9. Conclusions

Two finite element modelling techniques were applied to precisely capture the stresses corresponding to different fatigue crack types in slender curved steel girders. It was shown that initial imperfections have a significant influence on web boundary stresses. Moreover, high-stress concentrations at the location of the web-to-stiffener connection plate indicate how web curvature and lateral bracing can lead to fatigue cracking. The preliminary results of the ongoing research are presented. Other parameters involving the fatigue behavior of curved girders such as radius of curvature, slenderness, and aspect ratio will be investigated in future phases of the research.

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