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Bracing requirements to improve system buckling of narrow girder systems

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Abstract
Recent research has demonstrated the propensity of I-girder systems with a relatively large length/width ratio (ie. narrow girder systems) to fail by a system mode of buckling that is not sensitive to the spacing between traditional torsional bracing systems. The mode is particularly common in bridge systems that make use of cross frames or plate diaphragms for bracing. Several field problems have occurred involving the system mode of buckling in recent years and design equations have been developed to determine system buckling resistance. However, accurate design methods to improve the buckling resistance have not been developed. This paper outlines a parametric FEA study focused on the design of a top lateral truss to adequately raise the buckling strength of the girder system. The requirements include three criteria for the bracing: (i) adequate stiffness to reach the design moment, (ii) adequate stiffness to control out-of-plane deformations, and (iii) the adequate strength, which is a function of the initial geometric imperfections. The current research focuses on the first two requirements. The computational studies were executed to investigate the stiffness and the strength requirements for narrow girder systems, for which the global lateral torsional buckling mode is often critical. The paper presents the results of the executed numerical research program and introduces improved mechanical models and design equations for the determination of the required stiffness.

1. Introduction
Lateral-torsional buckling (LTB) is a failure mode that involves lateral movement and twist of the cross-section. Although cross frames permit lateral movement of girders, they can provide effective bracing in multi-girder bridges if the braces are properly proportioned to control girder twist. Although girders are often considered braced at the cross frame locations, a number of problematic bridges have had issues in recent years with a so-called “system buckling” failure mode in which multiple girders buckle as a unit. A number of recent bridge failures have occurred due to improper bracing systems worldwide, for example: Marcy Pedestrian bridge in Marcy, NY (Mehri 2012), Florida, Y1504 bridge in Sweden (Alenius 2003), Rio Puerco River bridge in Albuquerque, New Mexico (Jáuregui 2014), 102nd Avenue Bridge in Edmonton, Canada. Several problems were also reported in closely-spaced two or three girder systems experiencing significant twisting during construction and hardening of the concrete deck, for

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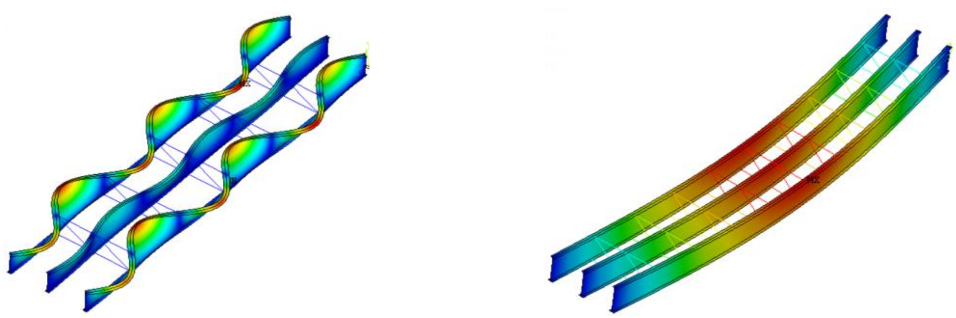
example as presented by Sanchez and White (2012). These reported problems call the attention on the importance of appropriate design of bracing systems and considering the global buckling failure mode in the design.

Effective stability bracing must possess adequate stiffness and strength. In the international literature beam bracing systems are usually separated into two general categories: (i) lateral bracing and (ii) torsional bracing (Yura 2001). The critical stage for stability in most bridges occurs during erection and construction when the steel girders alone generally support the applied loads. Based upon some of the previously cited failures, two general buckling modes that need to be considered include 1) lateral torsional buckling of the girders between the cross frame locations, and 2) system buckling of the girder system (mainly for “narrow” girder systems). These two failure modes are presented in Fig. 1. While designers are familiar with the LTB mode shown in Figure 1a, the current paper focuses on the system buckling failure mode and methods to improve the system buckling capacity.



a, LTB between torsional braces b, system buckling failure mode

Figure 1: Stability failure modes of girder systems.

Accurate design equations to determine system buckling capacity have been developed by Yura et al. (2008), which design model has been extended for uniformly distributed load and continuous girders by Han and Helwig (2019). The design equations allow an engineer to determine cases in which the system buckling mode controls the design; however they also need an effective and economical means of improving the system buckling capacity. There are currently two optional methods proposed by Yura et al. (2008) to increase the system buckling capacity: (i) increase the girder cross-sectional dimensions or (ii) apply additional lateral bracing. Because the system mode of buckling may only impact the behavior during construction, significantly changing the girder size or geometry may be inefficient. Instead, utilizing additional lateral bracing is often more efficient in terms of fabrication and material requirements. Yura et al. (2008) proposed preliminary expressions for the design of lateral bracing systems, however, some trial applications of the expressions have led to relatively large cross-sectional areas for the brace diagonals. To extend the preliminary work from Yura et al. (2008), a complex numerical research program was executed by the authors to determine the necessary lateral bracing requirement to increase the system buckling capacity of narrow girder systems. The current paper summarizes the results of the numerical parametric study investigating the requirements of a lateral truss bracing system. The current research program contains the followings main parts:

- investigation of the structural behaviour with and without a lateral truss for bracing,
- development of design equations for the critical buckling load of a braced system,
- determination of the stiffness requirements to control excessive out-of-plane deformations.

2. Literature review

2.1 System buckling capacity

An extensive numerical and analytical research program was previously executed by Yura et al. (2008) to investigate the structural behavior of narrow twin girders comprised of doubly- and mono-symmetric I-sections. The study provided an analytical solution to predict the critical buckling moment (M_{gs}) of two- and three- girder systems failing in the system mode of buckling. The study demonstrated that the buckling mode is relatively insensitive to the size and spacing of torsional braces. Two design equations (a complex and a simplified versions) were developed and presented in Yura et al. (2008) to determine the critical moment capacity. Due to the similarity between the resistance predicted by the simple and complex formulations, the simplified formula is the preferred expression and is therefore presented here in form of Eq. 1.

$$M_{gs} = \frac{\pi^2 \cdot S \cdot E}{L_g^{\,2}} \sqrt{I_y \cdot I_x} \qquad (1)$$

where: L_g is the span length, I_y is the effective out-of-plane moment of inertia, I_x is the in-plane moment of inertia, E is the Young's modulus of elasticity and S is the girder spacing. The above equation is valid for girders subjected by uniform in-plane moment, twist is restrained at the girder ends and warping deformations are unrestrained. The expression gives the combined strength of the girder system. The application range of this equation has been extended from the simply-supported systems to continuous systems by Han and Helwig (2019) in the form of Eq. 2. When the original equation from Yura et al. (2008) was incorporated in the US bridge specification (AASHTO 2015), the recommendation was to limit the application of the expression to cases where the design moment was less than or equal to 50% of the critical system buckling capacity due to the potential for second-order amplification. The recommendation was based upon each girder having the critical shape imperfection. Han and Helwig study demonstrated that considering the more likely imperfection of the girder system with bracing installed the limit on the expression could be raised from 50% to 70%. In addition, Han and Helwig recommended a moment gradient moment modifier for the system mode, C_{bs} for design (C_{bs} =1.1 for simple span or partially erected continuous span and 2.0 for fully-erected continuous girders). The resulting expression that is included in the current US Bridge specification (AASHTO 2017) is given in the following equation:

$$M_{gs} = C_{bs} \cdot \frac{\pi^2 \cdot S \cdot E}{L_g^{\,2}} \sqrt{I_y \cdot I_x} \quad ; \quad M_u \le 0.7 M_{gs} \qquad (2)$$

These design equations are considered currently as the most accurate equations to check the system buckling capacity of narrow girder systems. The authors checked their accuracy in frame of the current numerical parametric study and found good agreement with the FE results (all obtained differences to numerical investigations were smaller than 2%).

2.2 Bracing to improve system buckling capacity

In cases where the design moment exceeds 70% of the critical buckling capacity, the engineer must either alter the geometry (or increase the girder size) or introduce bracing. The most common form of bracing that can effectively improve the behavior is a top flange lateral truss. While a top lateral truss could be incorporated all along the length, the most effective locations will be regions in which the top flanges of the girders experience significant shear deformation. Therefore, this will generally occur near the support regions, as shown in Fig. 2.

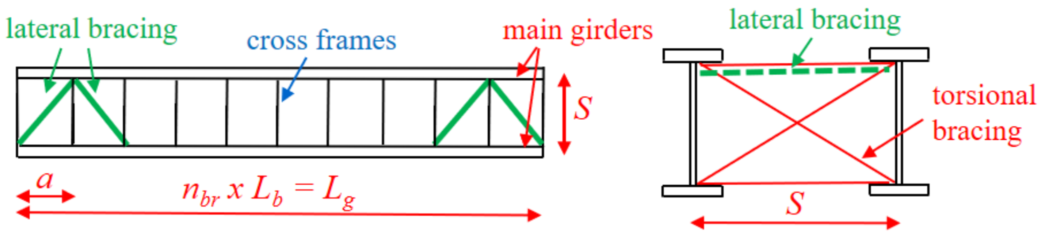


Figure 2: Stability failure modes of girder systems.

Two design methods using two different mechanical models are published in the international literature. The mechanical model shown in Fig. 3 and the proposed design equations are given by Eqs. 3-4 were provided in Yura et al. (2008). The mechanical model assumes that there are two rotational springs (warping constraints) at the girder ends, the stiffness of which can be calculated based on the cross-sectional area of the lateral bracing system and the braced panel geometries. Using these assumptions, the out-of-plane warping moment can be determined by Eq. 3, where M_u is the design moment requirement, M_{gs} is the global buckling critical moment without warping restraint (Eq. 2), L_g is the span length and h_0 is the cross-section depth of the girders. The warping moment can be transformed into a top flange shear and the required truss member cross-sectional area can be determined by using Eq. 4 for single diagonal system (Z-type lateral truss system formed by single diagonal and two struts), where L_w is the diagonal length of the truss system, a is the distance between cross-girders and A_d is the cross-sectional area of the diagonal members. This equation was derived from the torsional brace stiffness of a Z-type lateral bracing determined by Eq. 5 (Yura 2001).

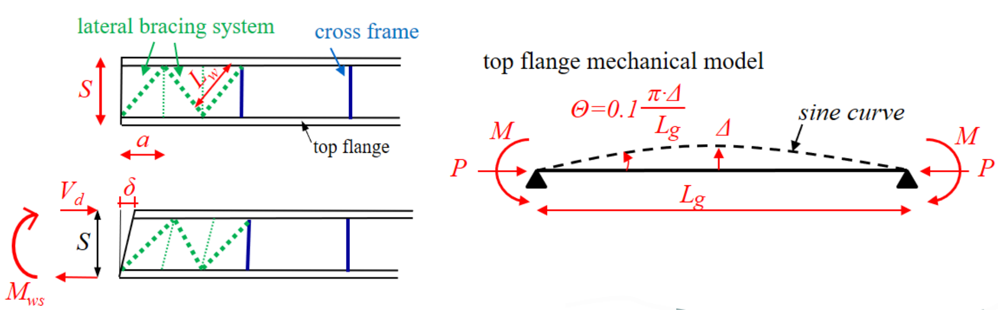


Figure 3: Mechanical model proposed by Yura et al. [8] for lateral bracing design.

$$M_{ws} = \frac{3 \cdot (M_u - M_{gs}) \cdot L_g}{h_0} \tag{3}$$

$$\frac{M_{ws}}{S^2} = \frac{\sum A_d \cdot L_b{}^2 \cdot E}{L_w{}^3 + S^3} \tag{4}$$

$$\beta_{b,unit} = \frac{A_d \cdot L_b{}^2 \cdot E \cdot S^2}{L_w{}^3 + S^3} \tag{5}$$

Second-order analyses of Yura et al. (2008) established that shear forces generated in the diagonals of the partial top lateral bracing system are related mainly to the magnitude of the initial out-of-straightness of the compression flange and the ratio of M_u/M_{gs}. However, the development of an exact analytical expression for the brace forces was beyond the scope of their paper. Therefore, as strength requirement, Yura et al. applied the 2% concept used for built-up column designs to evaluate the strength requirements for lateral braces according to Eq. 6, where F_d is the brace force in the diagonal member.

$$F_d = 0.02 \cdot \frac{M_u}{h_0} \cdot \frac{L_w}{a} \tag{6}$$

a in the above expression is depicted in Figure 3 and is the panel spacing. Another mechanical model for the same problem has been proposed by Mehri et al (2015). According to the proposed mechanical model the lateral truss system is supplemented by a discrete lateral spring with a stiffness k. The compressed flange alone is considered within this model laterally supported by two lateral elastic spring supports, as shown in Fig. 4.

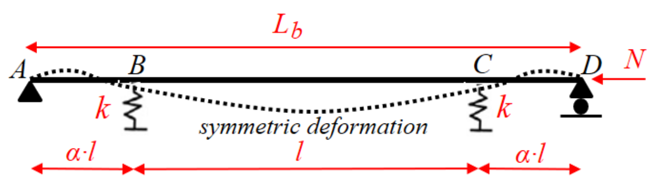


Figure 4: Mechanical model proposed by Mehri et al. for design of lateral bracing.

The spring position refers to the end of the braced part of the bridge. An analytical solution was derived for the out-of-plane buckling of the presented simple mechanical model, results of which are presented on design charts and diagrams. The executed research work has high potential in understanding the theory of lateral bracing of beams, however, the proposed design method is difficult to use in design problems.

Based on the literature review it can be concluded that there currently is no accurate mechanical model and design equations to determine the size and length of the required lateral bracing system for a given design moment. Therefore, the aim of the research investigation highlighted in this paper was to investigate the structural behavior of laterally braced girder systems to determine an appropriate mechanical model and to develop accurate, safe, and economical design methods for practical design. Further, the research aim is to consider the three general requirements for a narrow girder system with a partial top flange lateral truss: (i) critical system buckling moment capacity, (ii) resulting limitations to the out-of-plane maximum displacement and (iii) determination of the truss panel brace forces within one design method package to harmonize the stiffness and strength criteria for lateral bracing design.

3. Applied numerical models

3.1 Geometry and support conditions

Two individual, but identical numerical models were developed using the FE program ANSYS and ABAQUS to determine the critical buckling moment of girder systems with and without the lateral truss bracing. The numerical models consisted of four-node shell elements for upper and lower flanges, and web plate. Figure 5 shows the layout and boundary conditions of the twin-girder systems with torsional and lateral bracing.

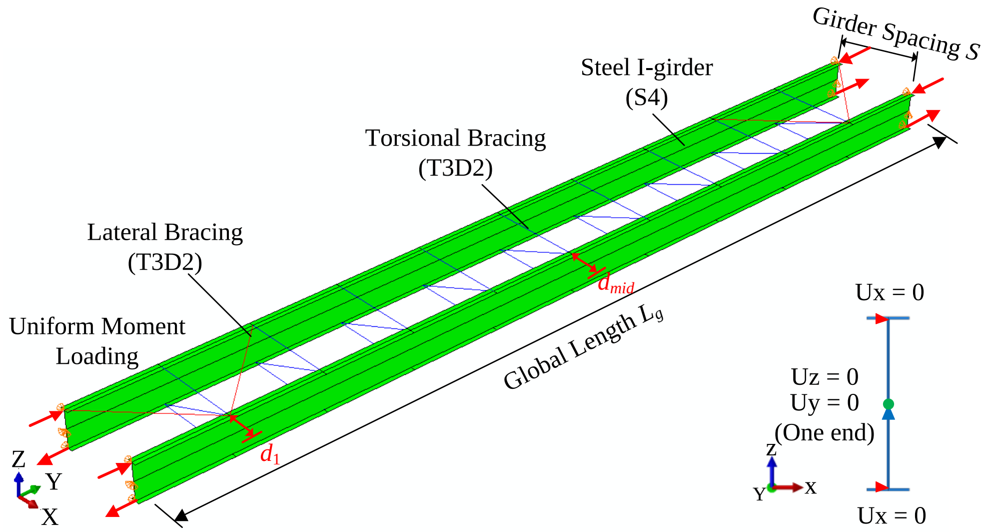


Figure 5: Geometry of the modelled structure and the numerical model.

The model was defined so that parametric studies could be carried out that contained at between 2 to 6 main girders subjected to uniform bending moment applied by force couples located at the two end cross-sections of the girders. The bracing system was modelled using truss elements. The truss elements that comprised the bracing system were connected to the web-to-flange junction points at the top of the girders. The girders were simply supported girders having fork-support conditions at the end cross-sections. The midheight node at each end of the beams were vertically and laterally supported, the middle node of the upper and lower flanges in the end cross-sections were laterally supported. The system was longitudinally supported at midspan in the middle point of the web. The first aim of the numerical modelling was to determine the critical buckling moment related to system buckling mode for various girder geometries with and without a lateral truss for bracing. The corresponding increase in the buckling capacity was determined for different bracing stiffness values of the lateral truss and the calculated critical moments were compared to previously proposed design methods. In the numerical parametric study for each analyzed girder geometry, the cross-sectional area of the lateral bracing system was increased to improve the critical moment resistance. Critical buckling moments were determined using bifurcation analyses on a linear elastic material model with a Young's modulus equal to 30000 ksi (210000 MPa). Following the eigenvalue analyses, geometrically non-linear analyses with elastic material models were carried out on a system with an initial imperfection. The applied imperfection shape is presented in Fig. 6, which is a sine shape where the peak sweep magnitude is equal by $L_g/1000$ located at the midspan. Although the critical shape of initial imperfections can be determined in many simple stability problems from the eigenvector; in bracing problems, such an approach does not work well. There have been a number of previous investigations on the critical shape imperfection by Wang and Helwig (2005). For torsional bracing, the critical shape has often been found to possess a lateral sweep of the compression flange while the tension flange remains straight. However, in the case of the system buckling mode, Han and Helwig (2019) observed that the shape of initial imperfection to be used is the lateral sweep of both flanges. The reason for this shape is that the "critical shape" with the top flange swept and the bottom flange straight requires the cross frames to be distorted. In reality, the cross frames are detailed and fabricated with relatively tight tolerances that tend to force more of a pure sweep on the multi-girder systems. Based on the non-linear analysis the load - out-of-plane deformation diagram of the girder system is determined. The maximum out-of-plane deformation and accompanying internal force in the bracing system is determined and their values at the target moment level are evaluated.

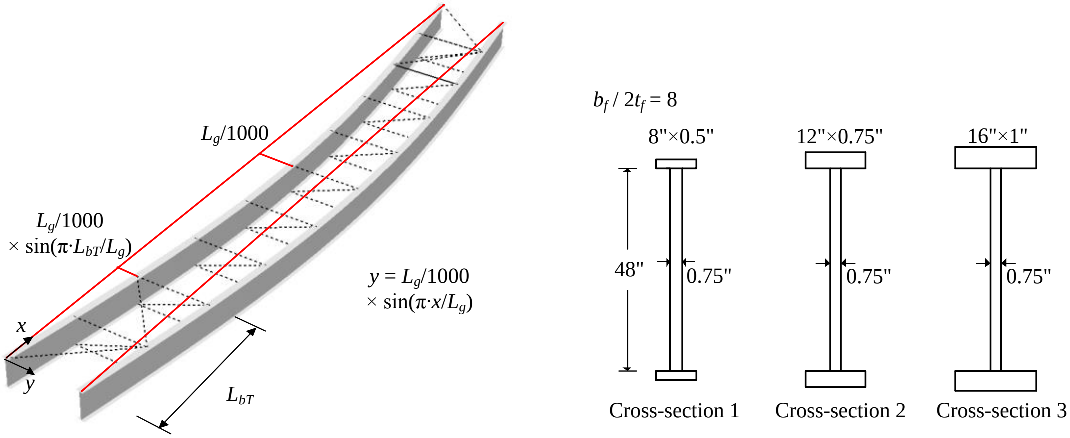


Figure 6: Initial imperfection shape and investigated cross-sections in non-linear analysis.

3.2 Investigated geometrical parameters

In the numerical parametric study, girder geometries that are commonly used in bridges were investigated. Table 1 summarizes the applied parameters used for the bifurcation analyses. A total of 398 different girder system geometries (different cross-sections, girder numbers, brace numbers or bracing lengths) were investigated in the eigenvalue analyses to study the stiffness requirements for the top lateral truss bracing system. For each analyzed girder system the lateral bracing stiffness was increased from 0 to reach at least $3 \cdot M_{gs}$. Although the maximum moment value of $3 \cdot M_{gs}$ is somewhat arbitrary, the researchers selected this value since it would likely represent an upper limit of the design moments during construction. In many cases, a value of $(1.5 – 2) \cdot M_{gs}$ is likely adequate.

Table 1: Investigated parameters and applied values in eigenvalue analysis.

parameter	applied values
h_w – web depth	3.3 – 3.9 – 4.6 – 4.9 – 5.2 – 5.9 [ft]
t_w – web thickness	0.8 – 1.0 – 1.2 [in]
b_f – flange width	6.0 – 7.9 – 9.8 – 11.8 – 13.8 – 15.7 [in]
t_f – flange thickness	0.8 – 1.0 – 1.2 – 1.4 – 1.6 – 2.0 [in]
a – distance between torsional braces	9.8 – 11.5 – 13.1 – 16.4 – 19.7 [ft]
n – number of torsional stiffeners	3 – 5 – 7 – 9 – 11
n_g – number of I-girders	2 – 3 – 4 – 5
n_{br} – number of braced panels	1 – 2 – 3 – 4 – 6 (refers to 0,05 L_g – 0,3 L_g)
L_g – total length (span)	80 – 200 [ft]

In the non-linear analyses, three prismatic cross-sections with flange widths of 8 in., 12 in. and 16 in. were considered, as presented in Fig. 6. The thicknesses of the flanges were selected to provide a width/thickness ratio of 8, which is compact for the commonly used grade 50 steel and therefore avoided local flange buckling. The flange sizes of the three sections provided flange-width-to-depth ratios of 1/6, 1/4, and 1/3. The value of 1/6 is the extreme limit for built-up sections allowed in the AASHTO bridge specification (AASHTO 2017), while the value of 1/3 is consistent with many rolled sections. The web thickness was chosen as 0.75 in. (19.05 mm) to maintain a relatively stocky web and avoid web local buckling so that the focus was on LTB and system buckling. Table 2 shows the parameters considered in the parametric study, including flange size, girder spacing S, number of lateral braces n_{br}, stiffness ratio of lateral bracing A_d/A_{di}, and applied moment level M_u/M_g.

Table 2: Investigated parameters and applied values in non-linear analysis.

Flange size	8"×0.5", 12"×0.75", 16"×1"
S	5', 10', 15'
n_{br}	1, 2, 3
A_d / A_{di}	1, 2, 3
M_u / M_g	1.4, 1.7, 2.0

3.3 Validation of the numerical model

A mesh sensitivity analysis was carried out to verify the numerical model and ensure adequate accuracy. The results of the mesh sensitivity study demonstrated that a finite element size of 1/20 of the web depth resulted in accurate critical moment values compared to analytical solutions for the lateral torsional buckling capacity of the girder between the brace points. The mesh density was generally larger than necessary, particularly for elastic analyses; however, researchers wished to ensure sufficient accuracy. The computed critical moment values were compared with the classic analytical solution of Timoshenko and Gere (Timoshenko 1961) and based on independent software tool (LTBeam) using 7 DOFs beam elements. Following the sensitivity study, the numerical model was validated by comparison of the numerical results with the system buckling critical moment from to Eq. 1. This equation was previously shown to have good accuracy by numerous researchers in the past studies. The comparison of the numerically and analytically calculated critical moment values showed good agreement, therefore the numerical model was considered validated and verified.

4. Mechanical model and design method for stiffness requirement

4.1 Results of the numerical parametric study

The length of the braced section of the beam is a function on the number of bracing panels (n_{br}) and the distance between torsional braces (a). The ratio of the braced section (n_{br} x a) and total length (L_g) are important parameters related to the structural behaviour. Therefore, systems with one, two or three braced panels were investigated and the relevant M_{cr}-$\beta_{b,unit}$ diagrams were studied. One example is presented for a twin girder system (h_w=3.9 ft; t_w=0.79 in; b_f=12 in; t_f=1.2 in; S=9.8 ft, L_b=9.8 ft) in Fig. 7. The vertical axis presents the calculated critical moment related to system buckling mode. The horizontal axis shows the brace stiffness ($\beta_{b,unit}$) calculated according to Eq. 5. Results show that increases in the number of bracing panels (n_{br}) results in an increase in the critical moment capacity of the system and the efficiency of lateral bracing. Cases were studied with lateral trusses on top flange (upper) and cases with lateral truss on both the top and the bottom flange (upper and lower). Figure 7 shows that the application of upper and lower bracing has no significant increase in the critical moment capacity compared to the case if only upper bracing is applied. The maximum difference between the two calculations is 15-20% considering all the three brace lengths. Considering that application of upper and lower bracing needs double the material and fabrication costs, it can be concluded that bracing only at the compressed flange zone is recommended in the design. Therefore, in the following numerical parametric study lateral bracing was always placed only in the plane of the compressed flange. Additional work is necessary to consider the behavior of continuous girders since the deformational effects may be significantly different in the negative moment region.

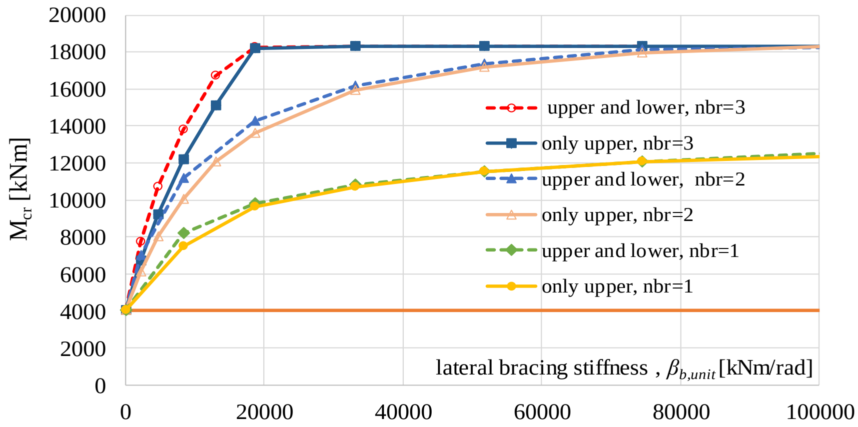


Figure 7: Effect of different bracing numbers and positions.

An extensive numerical parametric study was executed to investigate the effect of girder spacing, braced panel length, number of braced panels (number of diagonals), total girder length, effect of cross-section geometry (in-plane and out-of-plane inertia) and number of main girders. Due to length restrictions, all the results can obviously not be presented here, only a summary on the obtained trends are given. Results of the numerical study showed that the length of the region of the braced panels has a major effect on the critical moment capacity, the trends were almost linear. The number of braced panels has no direct impact on the lateral bracing efficiency. Cases with 2 larger panels versus 4 smaller panels with the same length of the region of bracing panels showed equal efficiency. The numerical parametric study also showed that the overall girder length has a significant effect on the lateral bracing efficiency; the observed trend was highly non-linear. Results also showed that the efficiency of the lateral bracing depends on the girder spacing as well. Girder systems with larger spacing requires smaller lateral bracing. The in-plane inertia has also a significant effect, while out-of-plane inertia has only a slight impact. Girder systems with larger values of the in-plane inertia require smaller lateral braces. The opposite trend is true for the out-of-plane inertia, which increases the necessary lateral bracing.

4.2 Proposed mechanical model

Taylor and Ojalvo (1966) investigated at first the effect of torsional bracing on the critical load amplifier related to lateral torsional buckling of elastically supported girders. An exact solution has been developed for the critical moment of continuously elastically supported I-girders in form of Eq. 7.

$$M_{cr} = \sqrt{M_{cr,0}^{\ 2} + \beta_{T,cont} \cdot E \cdot I_y} \tag{7}$$

where $M_{cr,0}$ is the critical moment of the girder without torsional springs, I_y is the out-of-plane second moment of area of the entire section and $\beta_{T,cont}$ is the specific stiffness of the continuous torsional springs. The accuracy of this equation has been shown by many researchers in the past and it has been proposed to be used for discrete springs as well (Yura 2001). Yura et al. (2008) proved that this mechanical model works for system buckling capacity as well. Although the present study is not focused on torsional bracing systems, the form of Eq. 7 provides a good working expression for investigating the behaviour of girder systems with a top lateral truss.

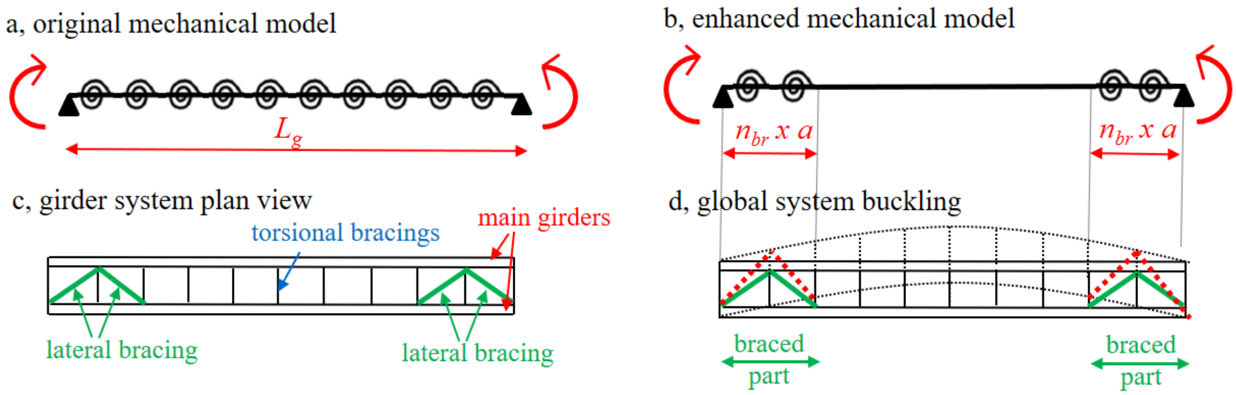


Figure 8: Proposed mechanical model considering lateral bracing.

However, lateral bracing is usually applied only at the two ends of the girder system, therefore the mechanical model should be enhanced according to Fig. 8b. In this mechanical model each braced panel is related to a discrete individual spring with stiffness ($\beta_{b,unit}$) as calculated by Eq. 5. The discrete spring can be transferred to a continuous spring divided by the panel length (a). Lateral bracing is not a continuous elastic support along the whole girder length, therefore its length has to be considered in the modified equation. The total length of the bracing panels can be considered in this mechanical model by $2 \cdot n_{br} \cdot a$ (lateral bracing is applied at two ends), and this length should be divided by the girder length (L_g). Additional parameters found to have an impact on the system buckling capacity (n_g, S/L_g, I_x, I_y) can be implemented into a unitless factor C_f, which is calibrated based on the results of the numerical parametric study. The general layout and mechanical background of the proposed new design equation is given by Eq. 8.

$$M_{cr} = \sqrt{\underbrace{M_{gs}^{\,2}}_{\substack{\text{system buckling}\\ \text{capacity without}\\ \text{lateral bracing}}} + \underbrace{C_f}_{\substack{\text{modification factor}\\ \text{to be calibrated}}} \cdot \underbrace{\frac{2 \cdot n_{br} \cdot a}{L_g}}_{\substack{\text{length of}\\ \text{bracing}}} \underbrace{\frac{\beta_{b,unit}}{L_b}}_{\substack{\text{bracing}\\ \text{stiffness}\\ \text{for unit}\\ \text{length}}} \cdot \underbrace{E \cdot I_y}_{\substack{\text{girder}\\ \text{stiffness}}}} \tag{8}$$

One example is presented in Fig. 9 demonstrating the comparison of numerical and analytical results for a twin-girder system with two braced panels (braced length of 0,2 L_g) and S/L_g ratio equal to 0,1. Numerical analysis proved that the proposed mechanical model is capable to predict the critical moment capacity increase coming from lateral bracing. The proposed mechanical model can be applied with large accuracy after calibration of the parameter C_f.

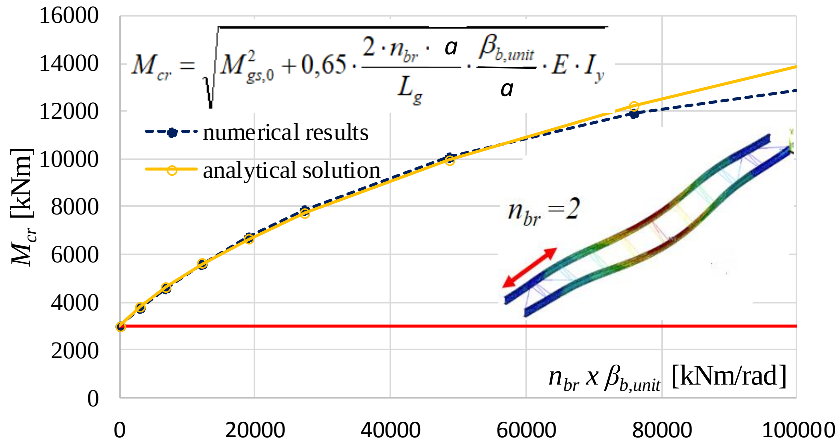


Figure 9: Comparison of numerical and analytical based $M_{cr} - \beta$ diagrams.

4.3 Proposed design model

The full 398 analyzed different girder systems (different cross-sections, girder numbers, brace numbers or bracing lengths) were evaluated in a similar fashion as shown in Fig. 9 and the necessary value of parameter C_f was determined. Numerical results showed, that this value should be smaller if the L_g/S ratio or I_y/I_x ratio is larger. Results for all analyzed girder systems are presented in Fig. 10 and the equation of the lower bound curve is given by Eq. 9.

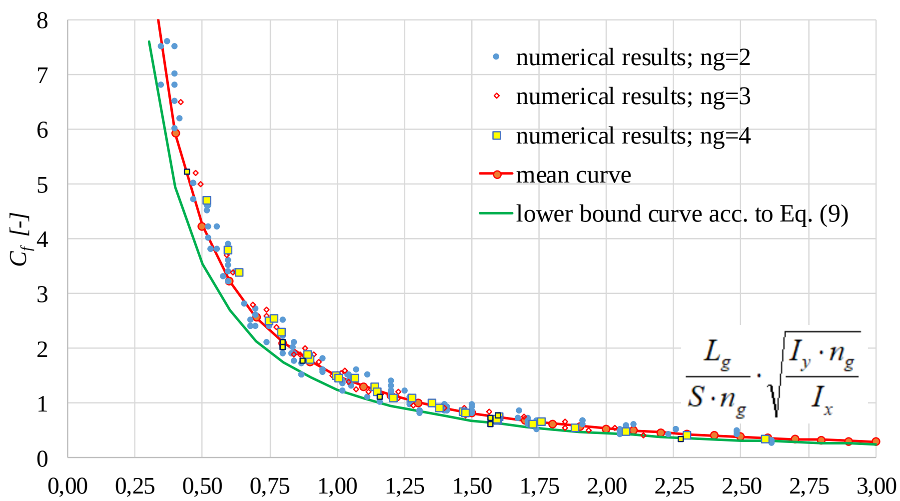


Figure 10: Calibration of parameter C_f.

$$C_f = \frac{1.25}{\left(\frac{L_g}{S \cdot n_g} \cdot \sqrt{\frac{I_y \cdot n_g}{I_x}}\right)^{3/2}} \tag{9}$$

Despite the previously presented equation is accurate, but due to its complexity it is hard to apply in design praxis. Therefore, it is simplified for design purposes. The simplified design equation is derived from Eqs. 8-9, and it is given by Eq. 10.

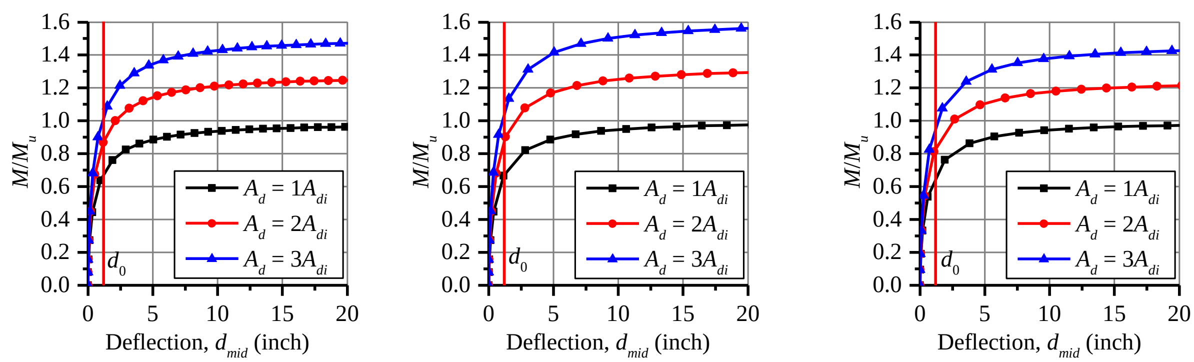
$$M_{cr.simplified} = \sqrt{\left(\frac{M_{gs}}{n_g}\right)^2 + n_{br} \cdot \beta_{b.unit} \cdot \frac{S^2}{L_g^{\ 3}} \cdot E \cdot I_x \cdot n_g^{\ 2}} \tag{10}$$

Implementing Eq. 5 into Eq. 10 the equation can be written in form of Eq. 11. This equation is similar to the design equation of Yura et al. (2008) given by Eqs. 3-4. However, it gives a better approximation of the critical moment capacity and the required lateral bracing stiffness.

$$\frac{M_{cr.simplified}^{\ 2} - \left(\frac{M_{gs}}{n_g}\right)^2}{E \cdot I_x \cdot n_g^{\ 2}} = \frac{\sum A_d \cdot a^2 \cdot E \cdot S^4}{L_w^{\ 3} \cdot L_g^{\ 3}} \tag{11}$$

5. Strength requirement and out-of-plane deformation limitation

For the development of strength requirement and deformation limitation geometrically nonlinear analyses were executed on numerical models containing imperfections. Bending moment and out-of-plane deformation diagrams were determined and evaluated for each analyzed case. The approach to control brace forces in many stability bracing formulations is to determine the stiffness necessary so that the additional deformation under load in the primary member is equal to the magnitude of the initial imperfection. Figure 11 shows the out-of-plane deflections at the mid-span for three specific cases having different bracing stiffness. In the graphs the vertical axis is normalized by the target buckling moment (M_u). The vertical red solid lines represent the deflection that equals the initial imperfection (L_g/1000). Three curves with different A_d/A_{di} ratios (A_d/A_{di} =1, 2, and 3) are compared on all the three diagrams. Although the applied moments approach the target moment by providing the ideal stiffness (A_d – $1 \cdot A_{di}$) for lateral braces, the corresponding deflections are considerable and sometimes more than 20 inches. By increasing the brace stiffness to two or three times the ideal stiffness, the normalized moments increase and the lateral deflection are significantly reduced. Results of the numerical parametric study proved employing three times the ideal area for lateral braces, the out-of-plane deformations at M/M_u=1 are close to the target value of the current study (initial imperfection d_0 – presented by red vertical lines in Fig. 11. Therefore, design recommendation of the authors is that three times the ideal area should be applied for lateral braces.



Sec.1, n_{br}=2, S=5’, M_u=1.7M_g Sec.2, n_{br}=2, S=5’, M_u=1.7M_g M_u=1.4M_g, n_{br}=2, Sec.2, S=5’

Figure 11: Effect of A_d/A_{di} on lateral deflection d_{mid}.

6. Conclusions

Numerical research program is executed to investigate the effect of lateral bracing system on the global system buckling behavior of girder systems. Based on the executed numerical calculations and theoretical considerations the following conclusions are drawn:

- each panel with lateral bracing can be considered as a discrete torsional spring with a spring stiffness ($\beta_{b,unit}$) and the mechanical model of elastically supported beams with continuous springs can be applied to consider their effect on the critical moment capacity,
- improved mechanical model and design equations are proposed to calculate the critical moment capacity of laterally braced systems and to determine the stiffness requirement,
- nonlinear numerical analyses showed that application of three times the ideal area for lateral braces would keep the lateral deflections on the target moment level close to the initial imperfection ($d_0=L_g/1000$).

The proposed design equations are valid and applicable only if global system buckling governs the failure mode of the analyzed girder and in case if target load level does not exceed $2 \cdot M_{gs}$. The current research program focused on the basic loading situation of LTB problems, uniform bending moment applied on simple supported girder. Further research is needed to check the applicability of the proposed design model for uniformly distributed load and continuous girders.

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References

AASHTO (2017) "AASHTO LRFD Bridge Design Specifications." 8th Edition. Washington, D.C.

ABAQUS (2017) Version 2017 Dassault System, USA.

Alenius, M. (2003) "Finite element modelling of composite bridge stability", MSc Theses, Royal Institute of Technology, Department of Mechanics, Stockholm, Sweden.

ANSYS® v16.5, Canonsburg, Pennsylvania, USA.

Han, L., Helwig, T.A. (2019) "Elastic Global Lateral-Torsional Buckling of Continuous I-shaped Girder Systems", (submitted for publication).

Jáuregui, D.V. (2014) "Evaluation of a Steel Girder Bridge Collapse during Deck Removal" *Journal of Performance of Constructed Facilities*, 28 (4).

LTBeam, Centre Technique Industriel de la Construction Métallique (CTICM), Saint-Aubin, France (V.1.0.11).

Mehri, H., Crocetti, R. (2012) "Bracing of steel-concrete composite bridge during casting of the deck", *Proceedings of the Nordic Steel Construction Conference*, Oslo, Norway.

Mehri, H., Croetti, R., Gustafsson, P.J. (2015) "Unequally spaced lateral bracings on compression flanges of steel girders", *Structures* 3 236-243.

Sanchez, T.A., White D.W (2012) "Stability of curved steel I-girder bridges during construction" *Transportation Research Record: Journal of the Transportation Research Board*, 2268 122-129.

Taylor, A.C., Ojalvo, M. (1966) "Torsional restraint of lateral buckling". *J Struct Div. ASCE,* 92(ST2) 115-129.

Timosenko, S., Gere, J., (1961) "Theory of elastic stability", McGrawe-Hill, New York.

Yura, Y.A. (2001) "Fundamentals of Beam Bracing." *Engineering Journal,* 11–26.

Yura, J., Helwig, T.A., Herman, R., Zhou, Ch. (2008) "Global lateral buckling in I-shaped girder systems" *J Struct Eng. ASCE,* 1487-1494.