Proceedings of the
Annual Stability Conference
Structural Stability Research Council
Atlanta, Georgia, April 21-24, 2020

Strength and stiffness requirements for beam torsional bracing

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Abstract
Effective torsional bracing for stability must satisfy both stiffness and strength requirements. The design philosophy for most stability bracing is to provide twice the "ideal" stiffness that often limits the member deformation to a value equal to the initial imperfection. For beam torsional bracing, computational studies have shown that the current AISC Appendix 6 provisions for stiffness do not meet this requirement. This paper outlines the results of a detailed numerical parametric study on the stiffness requirements for stability bracing to control the corresponding strength requirements. Previous research work has outlined the necessary "ideal" stiffness; however, some of the work has focused on single I-girders and the applicability of the developed design criteria has not been validated for multi-girder systems. Therefore, a numerical research program was executed to investigate the stiffness requirement of multi-girder systems. The results of the investigation highlight that the calculation method of the ideal stiffness related to single I-girders and multi-girder bridges are different and the application of design equations developed for single I-girders can lead to significant overestimation of the required ideal stiffness for twin or multi-girder bridges. However, the increment on the ideal stiffness to control deformations is often larger than twice the ideal stiffness. The paper presents result from the parametric studies and introduces improved design equations for the determination of the stiffness requirements to control brace forces and out-of-plane deformations.

1. Introduction
Torsional bracing is aimed to prevent twisting of the cross-section and improve lateral-torsional buckling (LTB) strength of steel I-girders (Fig. 1). Bridge girders with torsional bracing are usually designed assuming that buckling length is equal by the distance between the brace points. To ensure this assumption the bracing system should have enough stiffness and strength. Therefore, stability bracings have to fulfill both stiffness and strength requirements, which accurate determination is the topic of the current paper. The stiffness requirement has two issues: (i) to reach the critical moment level regarding fully braced girder (e.g. buckling length equal by the distance between the brace points) and (ii) to eliminate excessive torsion (or out-of-plane deformation) of the girders coming from geometric imperfections. The current research focuses on the stiffness requirement of torsional bracing and has the aim (i) to determine the required

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ideal stiffness of torsional bracing system, and (ii) to determine how many times the ideal stiffness is required to eliminate excessive out-of-plane deformation.

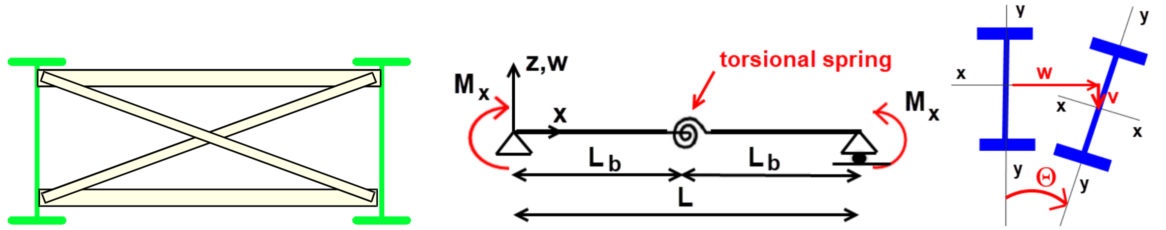


Figure 1: Example for torsional bracing (cross frame) and conversion its mechanical model to single I-girder.

The current AISC Specification (2010) recommends to calculate the required stiffness of torsional bracing by Eq. 1.

$$\beta_{T,req} = \frac{2.4 \cdot L \cdot M_r^{\ 2}}{\Phi \cdot n \cdot E \cdot I_y \cdot C_b^{\ 2}} \tag{1}$$

where $\beta_{T,req}$ is the total system torsional stiffness; L is the span length; M_r and n are the maximum factored moment and the number of intermediate bracing within the span; I_y is the effective out-of-plane moment of inertia of the I-girder; Φ is the resistance factor; C_b is the moment gradient factor; E is the Young's modulus of steel. This equation has been developed for design purposes to use double ideal stiffness in the design praxis to reduce brace forces and twist of the cross-section under construction. The theoretically derived ideal stiffness has been also multiplied by 1,2 considering top flange loading according to the investigations of Yura (2001).

Previous research results of Nguyen et al. (2010, 2012) highlighted that application of continous bracing theory can lead to significant differences compared to the required torsional stiffness of discrete braces. Numerical investigations proved that the current AISC Specification sometimes overestimates, sometimes underestimates the required ideal stiffness. Usual overestimation has been found for n=1 case (one mid-span bracing) and significant underestimation has been found for n≥3 cases (more than 3 bracings within one span). Due to criticism of the current AISC Specification a numerical research program has been conducted to investigate the differences between the continous and discrete bracings and their application in design equations. It is also observed, that all the previous investigations were focusing on single I-girders supported by torsional springs and its applicability has been checked for twin girders only for several cases. However, bracings in real bridges connect two or multiple girders which interaction can have significant effect on the structural behaviour and on the required torsional stiffness, which has not been investigated in the past studies. There are numerous previous investigations available in the international literature to determine the rotational stiffness of multi-girder bridges (β_{Tb}). It is known from international literature (Yura 2001; Helwig and Yura 2015) that there are three primary components of torsional bracing system stiffness that can have substantial impact on the structural behaviour: (i) brace stiffness (β_b), (ii) cross-sectional distortion (β_{sec}), and (iii) in-plane stiffness of the girders (β_g). The total torsional bracing system stiffness (β_{Tb}) can be calculated based on the stiffness of these three components according to Eq. 2.

$$\frac{1}{\beta_{Tb}} = \frac{1}{\beta_b} + \frac{1}{\beta_{\sec}} + \frac{1}{\beta_g} \tag{2}$$

However, how does relate the total torsional bracing system stiffness (β_{Tb}) to the ideal stiffness ($\beta_{T,req}$) needs more investigation. Authors realized that the stiffness requirement to avoid excessive torsion needs also revision, because its applicability is mainly proved for column buckling case and its validation is missing for lateral torsional buckling. Therefore, a large numerical parametric study is executed investigating the stiffness requirements of torsional bracings. Results on the numerical calculations are summarized in the current paper. The examined research program contains the followings main parts:

- comparison of structural behaviour of girders with continuous and discrete torsional supports and comparison to analytical solutions,
- numerical parametric study to investigate the critical moment of single I-girders and multi-girder bridges and to determine the required ideal stiffness,
- study and evaluation of the applicability of ideal stiffness calculation method for multi-girder bridges, their specialties are highlighted and design guidelines are proposed,
- determination how many times the ideal stiffness is necessary to eliminate excessive out-of-plane deformations.

2. Literature review

At first Taylor and Ojalvo (1966) investigated the effect of torsional bracing on the critical load amplifier and developed the exact solution for the critical moment of continuously elastically supported I-girders in form of Eq. 3.

$$M_{cr} = \sqrt{M_{cr,0}^2 + \beta_{T,cont} \cdot E \cdot I_y} \tag{3}$$

where $M_{cr,0}$ is the critical moment of the girder without torsional springs, I_y is the out-of-plane second moment of area of the entire section and $\beta_{T,cont}$ is the specific stiffness of the continuous torsional springs. The accuracy of this equation has been proved by many researchers in the past and it has been proposed to be used for discrete springs as well by Yura (2001) in form of Eq. 4. This equation has been implemented in the ASCI Specification (2010) in a simplified form.

$$\beta_{T,disc} = \frac{L}{n} \cdot \frac{\left(M_{cr}^2 - M_{cr,0}^2\right)}{E \cdot I_y} \tag{4}$$

Numerous another studies have been carried out on LTB of beams with discrete torsional bracing. Trahair (1993) developed an approximate equation for stiffness requirement of beams with mid-span torsional restraint (n=1 case). Valentino and Trahair (1998) performed investigations for beams with mid-span torsional bracing under various loading conditions. Valentino and Trahair (1998) studied the effects of mid-span torsional restraints on inelastic buckling by using the approximate solution proposed by Trahair (1966). Mutton and Trahair (1973), Nethercot (1973), Medland (1980), Tong and Chen (1988) have also studied the effect of torsional bracing on LTB strength of beams subjected by various loading conditions. However, almost all their studies are limited to mid-span torsional bracing problems. Significant research program has been performed by Nguyen et al. (2010, 2012) investigating the torsional stiffness requirement of discrete torsional bracing focusing on larger brace numbers (n≥1). Nguyen et al. (2010) proposed a design equation for the required torsional stiffness for discrete bracings, which follows the results of the numerical calculations for uniform bending moment. The applicability of the developed design equation has been extended by Nguyen et al. (2012) for uniformly distributed load as well. Mohammadi et al. (2016) also proved the results of Nguyen et al. and extended the design proposal to monosymmetric I-girders.

Previous research works made on twin-girder systems focused mainly on the global system buckling mode, as presented by Yura et al. (2008) and Han and Helwig (2019). It is important to separate the torsional stiffness requirement from the system buckling mode, because torsional braces are not effective to prevent global buckling. Similar works on inelastic buckling of torsionally braced I-girders under uniform bending has been made by Park et al. (2010) and Choi et al. (2010). They investigated multi-girder bridges using cross-beams, which structural behaviour and stiffness requirement is different from the requirements of cross-frames due to sensitivity for section distortion. However, one of the main conclusion of Park et al. (2010) was that the design equation proposed by Choi et al. (2010) gives the best proposal for the required stiffness of the cross-beam using $L/n+1$ instead of the previously proposed L/n ratio. Based on the literature survey the following conclusions are drawn:

- stiffness requirements developed for continuous elastic supports cannot be used for discrete bracings without modifications,
- previous studies highlighted that the current AISC Specification is on the safe side for $n=1$ and $n=2$ cases, however, unsafe for $n\geq3$ cases, which seems to contradict to the fact that $n\approx\infty$ case should be the closest to the analytical solution using continuous spring theory,
- stiffness requirements developed for single I-girders with discrete torsional springs cannot be applied for multi-girder bridges without modifications.

The aim of the current research program is to clarify the above mentioned contradictions, investigate the differences between continuous and discrete torsional bracings, single and multi-girder bridges and to develop simplified design equation for stiffness requirement.

3. Applied numerical models

3.1 Geometry and support conditions

Two individual, but identical numerical models are developed using FE program ANSYS and ABAQUS to determine the critical buckling moment of girder systems with different torsional bracing, to determine the ideal stiffness and the non-linear cross-sectional rotation during loading. The numerical models consisted of four-node shell elements for upper and lower flanges, and web plate. The general layout of the numerical model and the applied boundary conditions are presented in Fig. 2. Reasonable sized transverse stiffeners are also applied in the numerical model in the braced cross-sections to avoid section distortion.

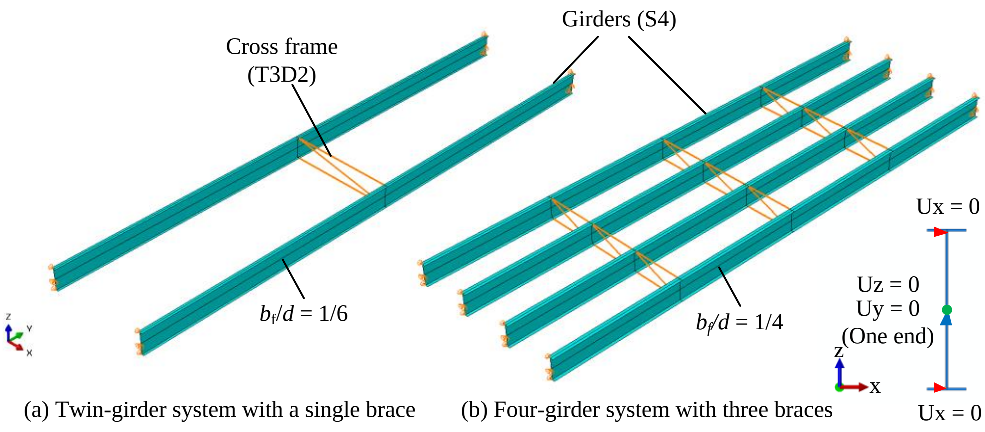


Figure 2: Parametric numerical model.

Parametric model is developed containing at least 1, maximum 6 main girders. Three loading conditions are applied and studied in the numerical parametric study as shown in Fig. 4: (i) uniform bending moment applied at the two end cross-sections, (ii) uniformly distributed load acting in the centre of gravity of the cross-section, and (iii) uniformly distributed load acting at the top flange. The bracing system is modelled using beam elements having pinned connections at the web-to-flange junctions. Applied support conditions refer to the simply supported single-span girders having fork-support conditions at the end cross-sections. The middle of each cross-section is vertically and laterally supported, the middle node of the upper and lower flanges in the end cross-sections are laterally supported. The system is longitudinally supported at midspan in the middle point of the web. The first aim of the numerical modelling is to determine the critical buckling moment related to system buckling for various girder geometries with different torsional bracing. In the numerical parametric study for each analyzed girder geometry the cross-sectional area of the bracing system is increased to increase the critical moment level. Critical moment is determined using bifurcation analyses using linear elastic material model with a Young's modulus equal to 30000 ksi (210000 MPa). Following the eigenvalue analyses, geometrically non-linear analyses with elastic material model and initial geometric imperfections is carried out. The applied imperfection shape is presented in Fig. 3, magnitude is equal to $L_b/500$. Wang and Helwig (2005) demonstrated that the critical shape of the initial imperfection for lateral-torsional buckling of beams consists of a lateral sweep of the compression flange while the tension flange remains straight. Figure 3 shows the asymmetric initial imperfection shape applied on the top flanges by displacing selected nodes laterally before the load application. The brace members were activated after applying the initial imperfection so that no forces were induced in the braces before the loading. Based on the non-linear analysis the load - out-of-plane deformation diagram of the girder system is determined. Maximum out-of-plane deformation and accompanying internal force in the bracing system is determined and their values at the target bending moment level are evaluated and discussed.

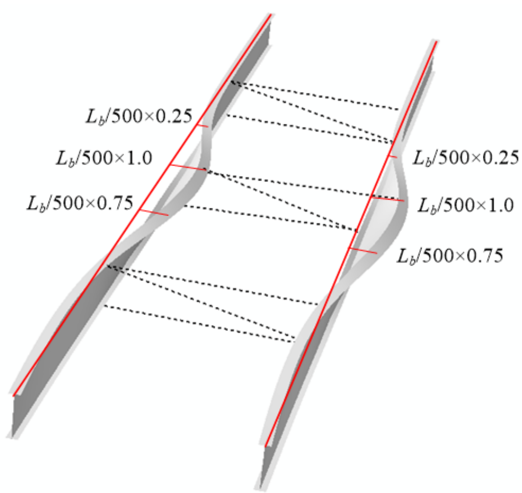


Figure 3: Initial imperfection shape used in the non-linear analysis.

3.2 Investigated geometrical parameters

Table 1 summarizes the applied parameters used for the bifurcation analysis. Total of 255 different girder geometries (different cross-sections and brace number or bracing lengths) are investigated in the numerical parametric study. For each analyzed girder the torsional spring stiffness is increased from 0 to reach the required critical moment level (parameter is varied between 0 - 2 times the required ideal stiffness).

Table 1: Investigated parameters and applied values in eigenvalue analysis.

parameter	applied values
h_w – web depth	2.6 – 3.3 – 3.9 – 4.6 – 4.9 – 5.9 – 6.5 [ft]
t_w – web thickness	0.6 – 0.8 – 1.0 – 1.2 [in]
b_f – flange width	7.9 – 9.8 – 11.8 – 13.8 – 15.7 [in]
t_f – flange thickness	0.8 – 1.0 – 1.2 – 1.4 – 1.6 [in]
$L/(n+1)$ – unbraced length	10 – 11.5 – 13 – 16.5 – 20 – 33 [ft]
n – number of stiffeners	2 – 3 – 4 – 5 – 6
L – total length (span)	33 – 100 [ft] (different values depending on $L/(n+1)$ and n)

In the non-linear analysis three prismatic cross-sections with the flange widths of 8 in., 12 in. and 16 in. are considered, as presented in Fig. 4. The thicknesses of the flanges are selected to provide a width/thickness ratio of 8, which is compact for the commonly used grade 50 steel and therefore avoided local flange buckling. The flange sizes of the three sections provided flange-width-to-depth ratios of 1/6, 1/4, and 1/3. The web thickness was chosen as 0.75 in. (19.05 mm) to maintain a relatively stocky web and avoid local buckling. The span of the girders was 100 ft., and the number of intermediate braces provided between the adjacent girders is 1, 3 and 5, as shown in Fig. 4. The corresponding unbraced lengths ranged from 50 ft. to 16.67 ft. To avoid issues with system buckling, the girder spacing ranged from 20 ft. to 30 ft. for the analyses investigating twin girder systems. This spacing was required for cases with five intermediate braces to prevent the buckling capacity from being controlled by the system buckling mode.

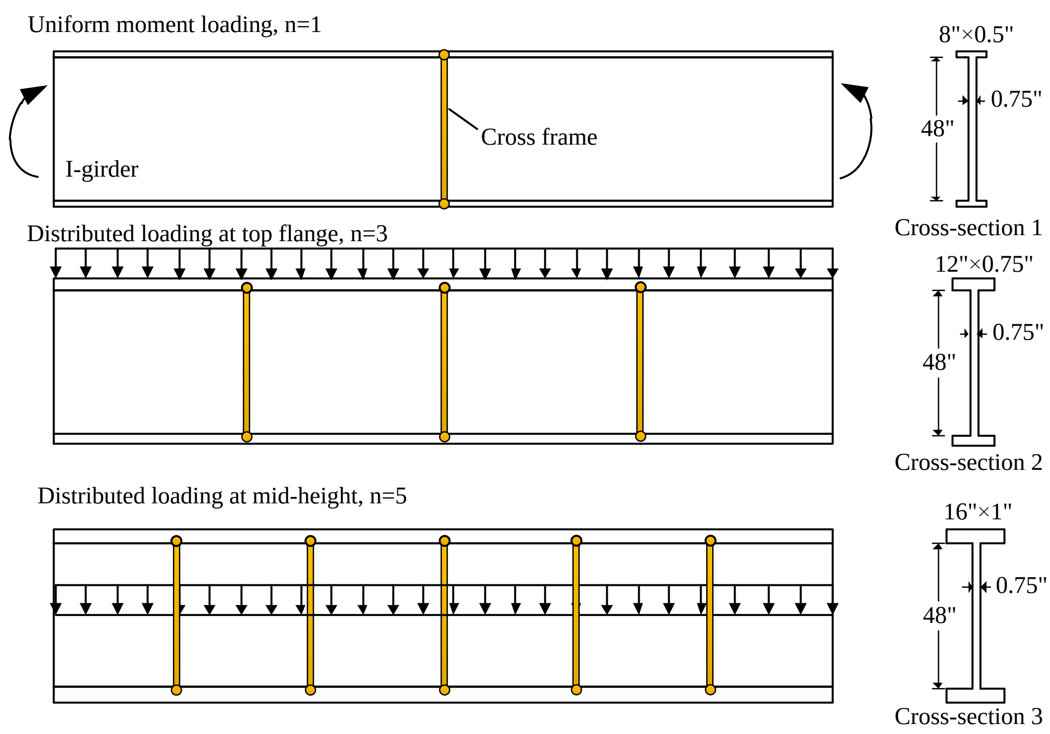


Figure 4: Initial imperfection shape used in the non-linear analysis.

3.3 Verification of the numerical model
Sensitivity analysis is conducted on two girders having the smallest and largest web depths (flange sizes as well) and the applicable element size is determined to ensure high accuracy. The result of the mesh sensitivity study proved that finite element size of 1/16 of the web depth results in accurate critical moments compared to analytical solutions and LTBeam results. Therefore, this finite element size has been used in the further studies for larger girder depths as well. After the sensitivity study the numerical model is validated by comparison of critical moment extreme values for each analyzed geometries with LTBeam and by hand calculation. It means that the numerical model is validated for each analyzed girder geometries for extreme values and the only running parameter for each girder is the stiffness of torsional bracing. For the extreme values the average difference between the numerical and reference model showed in average 1,5% difference. Therefore, the applied numerical model is considered to be verified.

4. Investigations on the ideal stiffness

4.1 Comparison of discrete and continuous spring supports
At first the structural behaviour of I-girders supported by discrete or continuous springs is investigated and compared. For all investigated girder geometries the spring stiffness is increased and the critical moment is determined. Results of the numerical calculations are plotted on M_{cr}-β diagrams. Three demonstrative examples are presented in Fig. 5 representing n=1, n=3 and n=5 cases loaded by uniform bending moment. Each diagram shows three calculation results, the dashed line presents the numerical calculations related to discrete springs, the orange line represents the results of a numerical model with continuous spring supports and the grey line shows analytical results using Eq. 3.

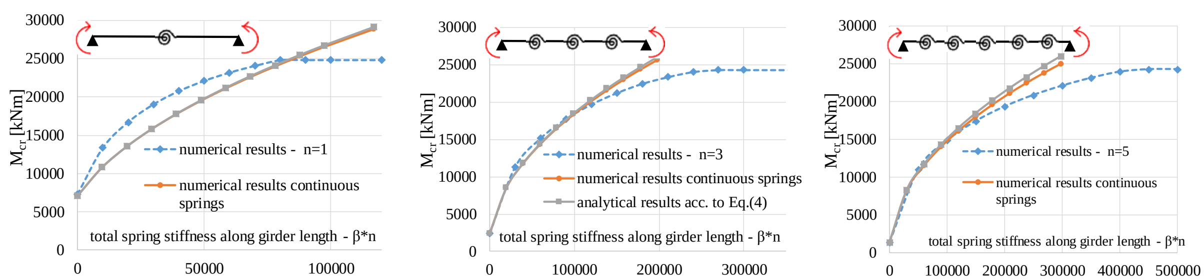


Figure 5: Differences in structural behavior between continuous and discrete springs.

The results proves the theoretical solution given by Eq. (3) gives extremely good agreement with the results of the numerical model using continuous springs. The behavior of the model using discrete springs significantly differs from the results using continuous springs, which proves that the efficiency of discrete and continuous springs is different. The theoretical solution using continuous spring theory gives the best approximation to the required ideal stiffness based on discrete spring model by n=1; the initial part of the diagrams show the best fit between the models using discrete and continuous spring by increasing the number of cross frames (n), which shows that by increasing the number of springs the structural behaviour of the model with discrete springs is getting closer to the analytical solution using continuous spring theory (which fits to the expectations). However, the efficiency of discrete springs are smaller by increasing their stiffness, therefore, larger ideal stiffness is needed using discrete springs than continuous springs.

4.2 Effect of total girder length and unbraced length (L or L/(n+1))

Because the efficiency of continuous and discrete springs is different, number of springs has an important role in the torsional stiffness requirement. Results of the numerical calculations show the required stiffness is not related to the number of springs, but related to the *L/(n+1)* ratio, representing the unbraced length. Figure 6 presents the results of numerical calculations and the analytical solution calculated by Eq. 4 using girders having constant total length (*L*), but different number of springs (*n*). Results prove that by increasing number of springs, critical moment increases and the required torsional stiffness increases as well. However, comparing the required values with the analytical solution it can be observed that the difference between analytical and numerical values is smaller for smaller spring numbers. This ratio increases by increasing number of braces and the analytical solution tends to be unsafe. However, if results are evaluated by keeping the unbraced length constant, as shown in Fig. 7, the required torsional stiffness does not change. It proves that the ideal stiffness should be not related to the numbers of torsional bracing, but to the unbraced length (which is apparently in relation with the brace number in case of practical cases, where the span is a given value and the number of bracing is a design parameter). It means that in the ideal stiffness equation *L/(n+1)* should be used to determine the required ideal stiffness instead of the currently used *L/n* ratio.

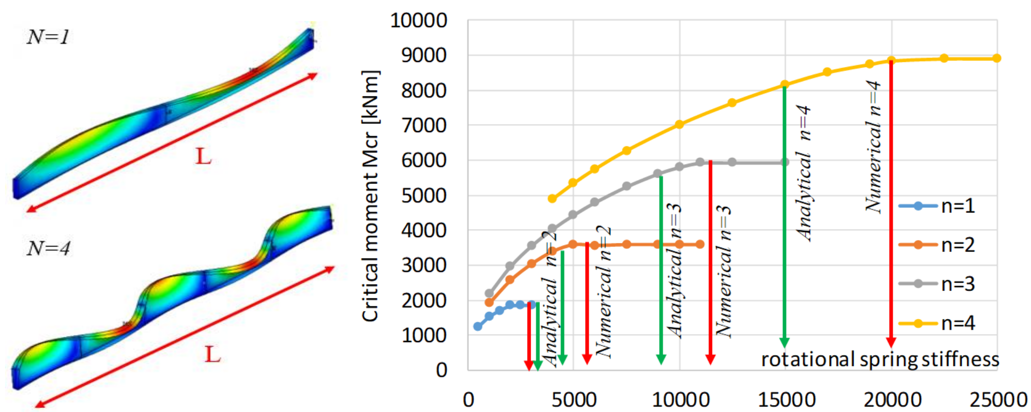


Figure 6: Numerical results for different number of spring with constant total length (*L*).

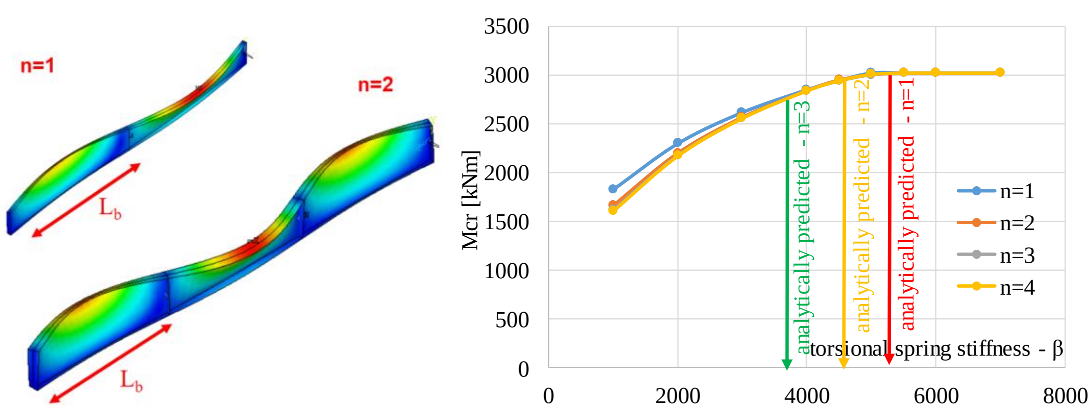


Figure 7: Numerical results for different number of spring with constant unbraced length.

Results also show, that the closest value to the numerical calculations is found for *n*=1 case, where the above mentioned change would halve the calculated ideal stiffness. Therefore, the required ideal stiffness has to be multiplied by 2, as given by Eq. 5. This modification leads to identical ideal stiffness values with the current specifications for n=1, but it could lead to significant differences for n>1 cases. Therefore, the design equation needs further improvement, another modification factor ($N_{mod,twin}$) considering the cross-section properties, number of torsional bracings, girder spacing and calculation method of the total torsional bracing system stiffness (β_{Tb}). Calibration of this modification factor is the task of the ongoing research work of the authors.

$$\beta_{T,ideal} = \frac{M_{cr,n+1}^{\;2}}{E \cdot I_y} \cdot \frac{L}{n+1} \cdot 2 \cdot N_{\mathrm{mod},twin} \tag{5}$$

4.3 Effect of girder number (n_g) on ideal stiffness

Numerical parametric study is also executed to check the effect of girder number (n_g) on the ideal stiffness. The typical M_{cr}-β_{Tb} diagrams are determined and compared for all analyzed girder systems. All M_{cr}-β_{Tb} curves are evaluated using the analytical equations given by Eq. 2. One representative example (n=1 case) is presented in Fig. 8, showing that there is a significant difference in the ideal stiffness depending on the number of girders. If n_g increases, the ideal stiffness decreases. The results show that twin-girder system approximates the ideal stiffness of single I-girder and the largest change is observed between n_g =2 and n_g =3. Results also means, that ideal stiffness developed for single I-girders or twin girders can conservatively be applied for multi-girder systems. However, an adjustment factor is necessary for multi-girder systems to ensure economic design.

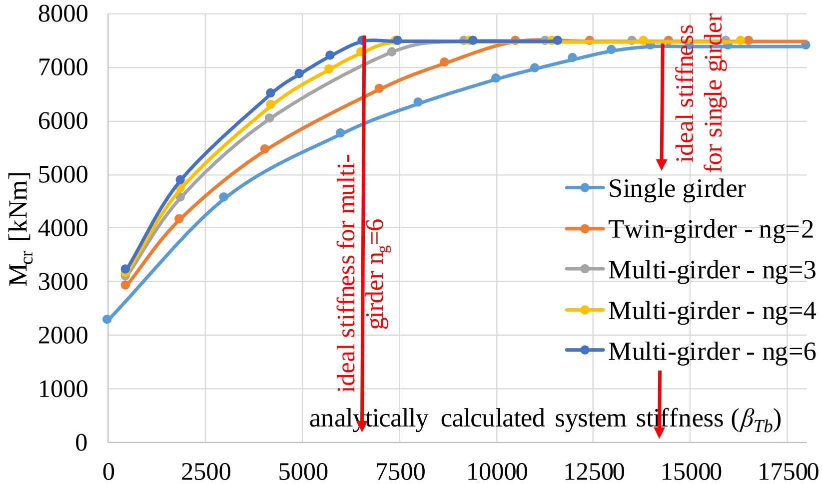


Figure 8: Comparison of M_{cr}-β_{Tb} diagrams – based on analytically calculated β_{Tb}.

To calibrate this modification factor similar diagrams as presented in Fig. 8 are produced and evaluated for all analyzed girder geometries. The ratio of ideal stiffness values calculated for twin and multi-girders are determined and presented in Fig. 9. Graph shows that the calculated values do not show large scatter and there is a clear trend between the obtained ideal stiffness and girder numbers. Based on the executed numerical calculations modification factor is developed for the ideal stiffness given by Eq. 6 presented by red line in Fig. 9.

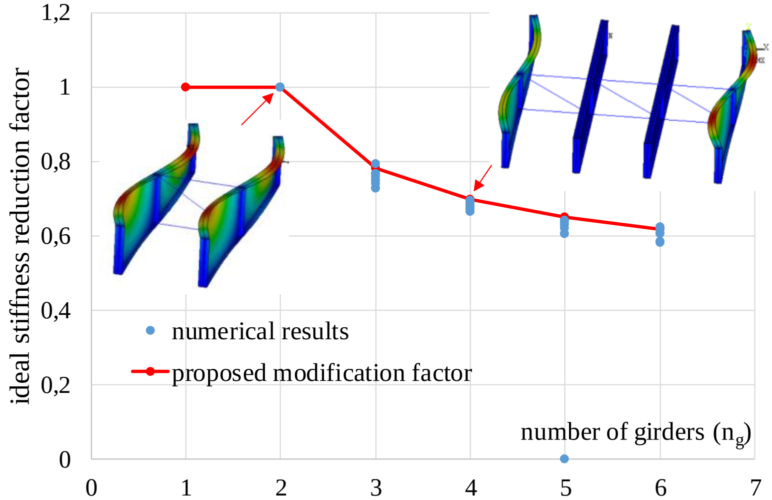


Figure 9: Ideal stiffness reduction factor depending on n_g.

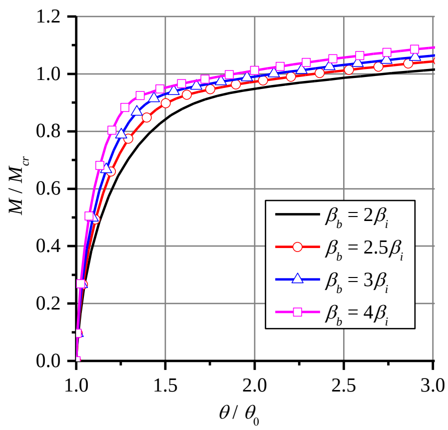
The modification factor can be applied together with all ideal stiffness calculation methods, because it follows the trend of the numerical simulations and it is derived based on the comparison of twin and multi-girder systems. The proposed equation keeps the accuracy of the ideal stiffness calculation method developed for twin-girders and makes it applicable for multi-girder systems ensuring the same accuracy.

$$N_{\mathrm{mod},g} = \frac{1}{n_g} + 0{,}45 \quad \text{if} \quad n_g > 2 \tag{6}$$

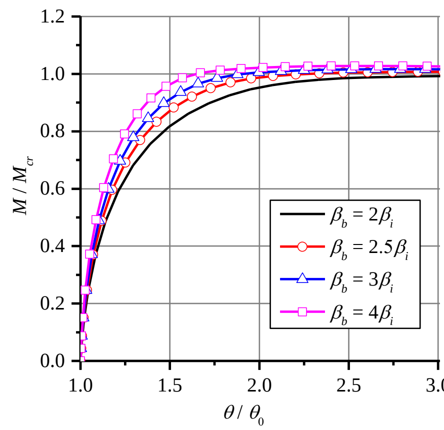
The above presented results are related to analytically calculated β_{Tb} values and verified for n=1 and n=3 cases. Further numerical investigations are needed to prove the accuracy of this equation for larger n values and using various cross-sections.

5. Investigations on the cross-section rotation

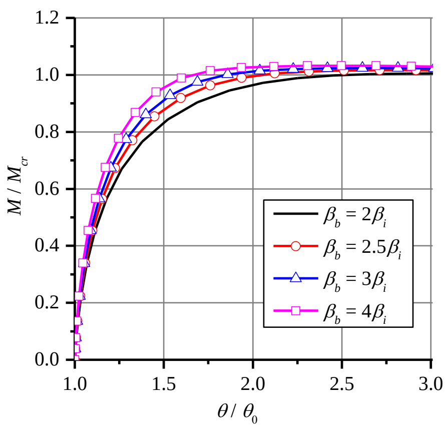
After determination of the ideal stiffness non-linear analysis are executed to determine the out-of-plane deformation of the braced cross section using imperfect numerical model and geometrically nonlinear calculation. Figure 10 shows graphs on the normalized bending moment versus the corresponding relative twists for the cases with the bracing stiffness ranging from two to four times the ideal stiffness ($\beta_{T,ideal}$). The applied moment for a single girder is the average moment among the girder systems and is normalized by the critical buckling moment. In the graphs, β_i and M_{cr} are obtained from the eigenvalue analyses; θ_0 is the cross-sectional twist induced by the initial imperfection. It can be observed the relative twists at M_{cr} or the maximum applied moment level are always higher than 2. It means the results showed in the figures are against the assumption that the cross-sectional twist equals to the initial imperfection by providing twice the ideal stiffness. By employing higher bracing stiffness, the slopes of the curves increase, and the relative twist at the limit state effectively decrease. Figure 10(a), (b) and (c) show the effect of the number of bracing on the stiffness requirement for torsional bracing by the cases with cross-section #2. The cross-sectional twists decrease by introducing more bracing. Figure 10(b), (d) and (e) compare the cases with three braces and different cross-sections. The results show that the twist occurs for cross-section #2 is lower than for cross-section #1 and #3. Figure 10(b) and (f) present the results of the cases under top flange loading and mid-height loading with three braces. In these cases, the transverse loading locations have a negligible impact on the sectional twist. Results of the numerical calculations show that by providing twice the ideal stiffness which agrees with the current assumption, the relative twists are always larger than 2. However, the sectional twist can be reduced to around the initial imperfection by using the braces with three times the ideal stiffness.



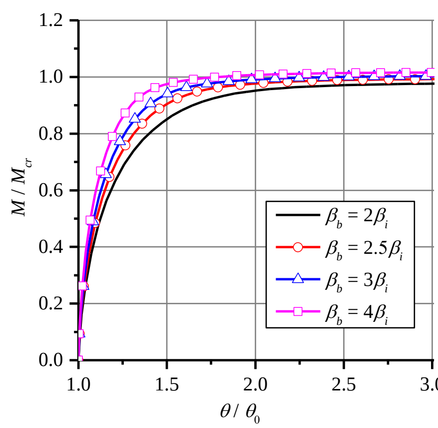
(a) n=1, Sec.2, top flange loading



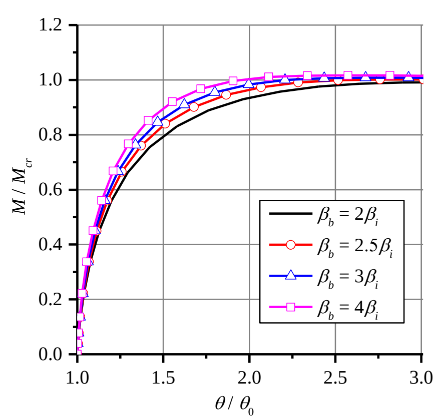
(b) n=3, Sec.2, top flange loading



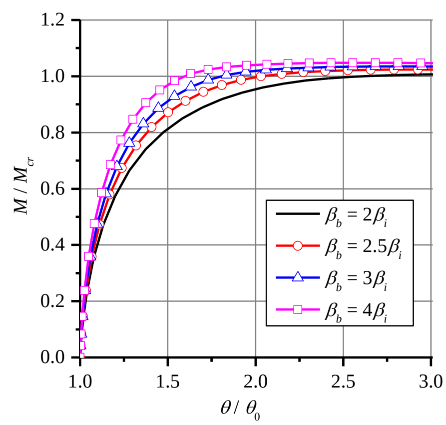
(c) n=5, Sec.2, top flange loading



(d) n=3, Sec.1, top flange loading



(e) n=3, Sec.3, top flange loading



(f) n=3, Sec.2, mid-height loading

Figure 10: Computed moment – rotation curves.

6. Conclusions

Numerical research program is executed to investigate the strength and stiffness requirement of torsional bracing systems. Based on the executed numerical calculations and theoretical considerations the following conclusions are drawn:

- effectiveness of discrete torsional springs is different from continuous spring, therefore the design equations using continuous spring theory should be modified,
- the required ideal stiffness is independent from the number of brace numbers, it depends on the unbraced length; therefore, $L/(n+1)$ should be used in the ideal stiffness calculation instead of L/n ratio,
- increasing the girder number (n_g) leads significant reduction of ideal stiffness; twin-girders require the largest ideal stiffness; reduction factor is to be applied for multi-girder bridges,
- twist can be reduced to around the initial imperfection by using three times ideal stiffness.

Acknowledgments

The authors would like to acknowledge *China Scholarship Council (CSC)* for the financial support (No. 201706260122) of Yangqing Liu and the *Korányi Research Scholarship* for the financial support of Balázs Kövesdi allowing them to participate in this research program at University of Texas at Austin under the supervision of Professor Helwig.

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