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The effect of shear coexistent with axial compression on transverse stiffeners in longitudinally stiffened plates.

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Abstract

Longitudinally stiffened steel plates are becoming increasingly important in bridge design as the size of new bridges increases. Examples are the steel spans (26ft wide box girders) of the Mumbai Trans Harbour Link, under construction in India, and the deck and towers of the Izmit Bay suspension bridge, recently completed in Turkey. The stability of wide plates with multiple longitudinal stiffeners depends on the transverse stiffeners to restrain the longitudinal stiffeners. The destabilizing actions on the transverse stiffeners include both the longitudinal compression and the in-plane shear. The design methods currently available in design codes account for the longitudinal compression in the plate but do not include the in-plane shear. Therefore, designers do not know if they should either ignore shear coexistent with compression or attempt to account for it.

This paper presents the results of finite element analyses showing the destabilizing effect of shear in addition to compression on the transverse stiffeners of longitudinally stiffened plates. It considers plates with different aspect ratios. The analyses use non-linear geometry with non-linear material properties and the plates are modeled with initial geometrical imperfections. The destabilizing effects of different magnitudes of in-plane shear coexistent with longitudinal compression are compared with the effects calculated using a simple analytical model suitable for use in a design office.

1. Introduction

The content of this paper has been developed primarily for the design of wide plates with multiple longitudinal stiffeners. The primary examples of these are towers of suspension bridges and cable stayed bridges and compression flanges of box beams, but the same principles apply to any member in which the compression resistance depends on the spacing of the transverse stiffeners to define the buckling length of the longitudinal stiffeners.

The stability of transverse stiffeners in longitudinally stiffened plates may be calculated by, Appendix E6.1 or by Eurocode 3 Part 1.5 (EN1993-1-5). The equations consider both stiffness and strength for the effects of longitudinal compression and compression in the transverse stiffener together with the initial imperfections. AASHTO LRFD 9th edition also presents equations for

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additional transverse loads on the transverse stiffeners. However, they do not account for the effects of coincident shear in the longitudinally stiffened plate. These effects include both (a) the reduced bending capacity in the transverse stiffeners due to the longitudinal compressive stress and the coexistent shear stress in the plate and (b) the additional destabilizing forces from the shear stresses in the plate.

This paper presents information to allow a designer to account for net transverse shear forces in wide plates in which the compression resistance depends upon the spacing, a, of the transverse stiffeners. It is written in terms of AASHTO LRFD 9th edition but the same approach can be used for other codes such as Eurocode 3 Part 1.5, EN 1993-1-5.

The approach is to find what compression and coexistent shear can be carried by a longitudinally stiffened plate in which the transverse stiffeners are sized to resist the maximum compression resistance, P_{nsp}, of the longitudinally stiffened plate.

The finite element analyses presented in this paper are for a plated structure with a length of several panels as shown in Figure 1. The dimensions are given in Table 1

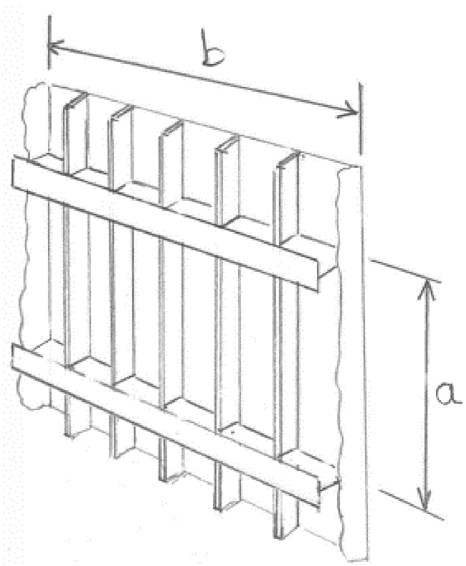

Figure 1: Longitudinally stiffened plate

Table 1, Dimensions of stiffened plates considered		
NOTE: Sizes in inches are approximate		
	3m spacing	6m spacing
Width of plate, b	8.0 metres (26 ft)	
Spacing of transverse stiffeners, a	3.0 metres (10ft)	6.0 metres (20ft)
Plate thickness	50mm (2 ins)	
Longitudinal stiffener	300×30 mm (12×1 3/8 ins)	462×40 mm (18×1 5/8 ins)
Transverse stiffener web	791×12mm (32×1/2ins)	603×12mm (24×1/2ins)
Transverse stiffener flange	400×20 mm (16×3/4 ins)	

2. Modifying code equations to account for coexistent shear

This is illustrated by two of the equations in AASHTO LRFD 9th edition for cases without either axial compression or lateral loads applied to the transverse stiffener:

the equation for stiffness,

$$I_t \geq 0.05 \frac{P_{up}}{a_{min}} \frac{b_{sp}^3}{E} \tag{1}$$

and the equation for strength

$$I_t \geq \left(0.0009 \frac{E}{F_y} \frac{c}{b_{sp}} + 0.02\right) \frac{P_{up}}{a_{min}} \frac{b_{sp}^3}{E} \tag{2}$$

These equations account for the longitudinal axial compression, P_{up}, the nominal yield stress, Fy, and the largest distance from the neutral axis to the extreme fibre. These equations are appropriate for the common cases in which the shear stresses are low and the neutral axis is much closer to the plate than to the outstand. They do not account explicitly for shear stress in the plate, nor do they account for the reduction in yield in the plate due to the coexistent shear and tensile stresses from bending of the transverse stiffener.

In common cases of transverse stiffeners, distance from the neutral axis of the stiffener to the extreme fibre of the outstand of the stiffener is much further than the distance from the neutral axis to the extreme fibre of the plate. Therefore, the bending moment causes the outstand of the stiffener to reach yield stress while the bending stress in the plate is low. However, if the shear stress in the plate is big, the plate may yield at lower bending moments than those causing yield in the outstand. The reduction is even greater at the stiffeners in which the bending causes tensile stresses in the plate.

For cases with significant coexistent shear in the plate, designers should replace the strength check equation above with two equations, one to check the outstand of the stiffener and one to check the plate acting as the flange of the stiffener. Both of these equations are similar to the strength check above.

To check the strength of the outstand of the stiffener:

$$I_t \geq \left(0.0009 \frac{E}{F_y} \frac{c_o}{b_{sp}} + 0.02\right) \frac{P_{upe}}{a_{min}} \frac{b_{sp}^3}{E} \tag{3}$$

where

c_o = distance from the neutral axis of the transverse stiffener to the extreme fibre of the outstand,

P_{upe} = longitudinal compression force giving destabilizing effects equal to destabilizing effects of the applied longitudinal compression and the coexistent shear.

To check the strength of the plate acting as a flange of the stiffener:

$$I_t \geq \left(0.0009 \frac{E}{F_{tr}} \frac{c_p}{b_{sp}} + 0.02\right) \frac{P_{upe}}{a_{min}} \frac{b_{sp}^3}{E} \quad (4)$$

c_p = distance from the neutral axis of the transverse stiffener to the extreme fibre of the plate,
F_{tr} = limiting value of transverse stress in the plate, reduced below the nominal yield stress.

3. Reduced yield stress at the transverse stiffener

At each transverse stiffener, the plate that is longitudinally stiffened forms a flange of that transverse stiffener. The plate will yield when the coexistent stresses satisfy the yield criterion. The von Mises yield criterion for 2-dimensional stress may be written as:

$$\sigma_y^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 + 3\tau^2 \quad (5)$$

where
σ_y = yield stress of plate
σ_1 = longitudinal stress in plate at the transverse stiffener
σ_2 = transverse direct stress in plate from action as the flange of the transverse stiffener
τ = shear stress in plate

Using the symbols in the strength check equations, the von Mises yield criterion limits the transverse stress in the plate so that

$$F_y^2 \leq \sigma_1^2 + F_{tr}^2 - \sigma_1 F_{tr} + 3{f_{ve}}^2 \quad (6)$$

where
f_{ve} = the coexistent shear stress in the stiffened plate

4. Magnitude of destabilizing effect of shear

The magnitude of the destabilizing effect of shear was investigated by the finite element analyses described below. These effects are in addition to any reduction in capacity caused by the reduced yield stress arising from the yield criterion. The destabilizing effect of the coexistent shear was found to increase as the ratio of shear/axial increases.

Expressing the destabilizing action of coexistent compression and shear in terms of an equivalent compression force alone, the equivalent compression may be defined as:

$$P_{upe} = P_{up} + P_v \quad (7)$$

where

$$P_v = C_{fv} \times b_{sp} \times t_{sp} \times f_{ve} \quad (8)$$

t_{sp} = thickness of the stiffened plate
f_{ve} = the coexistent shear stress in the stiffened plate

C_{fv} = coefficient to calculate equivalent longitudinal compression which increases with increasing shear stress.

The increasing trend of C_{fv} is shown in Figure 2.

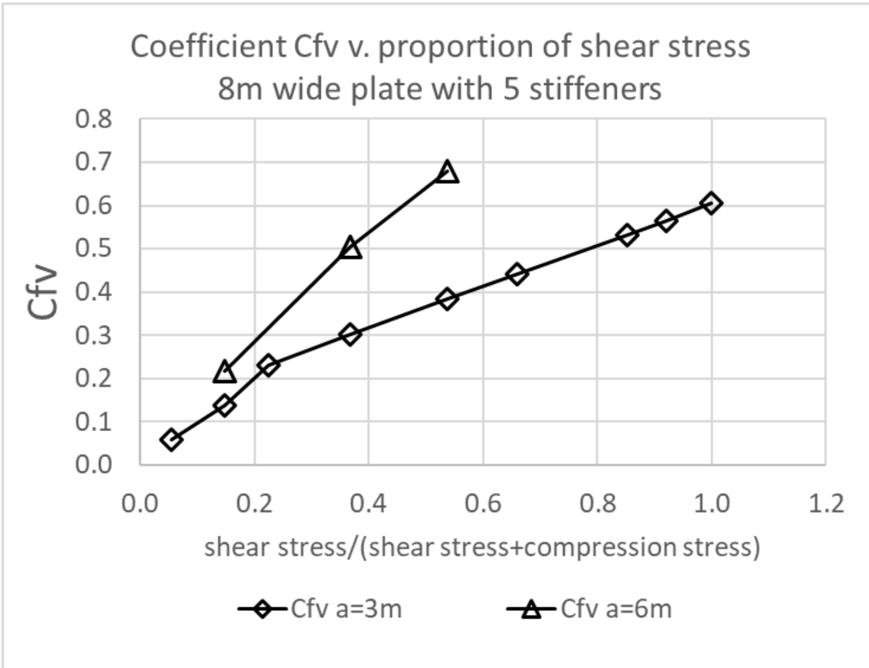

Figure 2: Destabilizing effect of shear

4. Finite element analysis

The structural behavior was studied using finite element models analysed with Abaqus software using shell elements. The models were run using non-linear geometry and non-linear material properties with initial imperfections and residual stresses. The residual stress patterns were taken as 0.3Fy in the longitudinal generally and 0.2Fy in the transverse stiffener webs and flanges because they are generally of thinner material. In addition to the effects of residual stresses, the stress-strain curve included an additional softening in the region of yield stress to achieve yield stress at 0.2% proof strain.

The first analyses were made with non-linear material in the plate, but, at high shear stress, this results in failure from the yield criterion which masks the destabilizing effect of the shear stress. Therefore, later analyses were conducted with elastic properties for the plate, so the destabilizing effect could be identified and the coefficient Cfv could be calculated. Non-linear material properties were retained for the longitudinal and transverse stiffeners and the transverse stiffener because the onset of yield severely reduces the stiffness of these stiffeners, thus having a significant effect on the stability of the entire stiffened plate.

5. Initial geometrical imperfections

The design value of the initial imperfection of transverse stiffeners in AASHTO LRFD 9th edition is $b_{sp}/250$ (White et al 2019) where b_{sp} is the width of the stiffened plate. In the legs of large towers of suspension bridges, the spacing of the transverse stiffeners is commonly much less than the width of the stiffened plate. A representative plate in the leg of a large tower is 8 metres (26 ft) wide with 5 longitudinal stiffeners and transverse stiffeners at 3 metres (10 ft) centres. Taking the initial imperfection as $b_{sp}/250$ gives a value of 32 mm (1 ¼ ins). This could lead to an unreasonably severe initial curvature to the longitudinal stiffeners where the transverse stiffener spacing is 3 metres, so the studies reported in this paper limited the imperfection of the transverse stiffeners to the lesser of $b_{sp}/250$ and 2a/250, where a is the spacing of the transverse stiffeners.

The specification for fabrication of stiffened plates should include tolerances on the straightness of transverse stiffeners and it is prudent to specify a slightly smaller tolerance than the design value. For example, if the design values are the lesser of $b_{sp}/250$ and 2a/250, the fabrication tolerance might be specified as the lesser of $b_{sp}/300$ and 2a/300.

In finite element analyses for the stability of line members, it is conventional to take the shape of the initial imperfections to be the same shape as the eigenvector for the elastic critical buckling of the member. In longitudinally stiffened plates with multiple stiffeners, this approach can lead to misleading results because it has been seen that the first mode in a plate panel may have one stiffener buckling in one direction and the adjacent stiffener buckling in the opposite direction. This is not consistent with the lowest failure of the whole plate panel, in which all stiffeners deflect in the same direction. Therefore, the finite element analyses reported in this paper used an initial imperfection form that has adjacent stiffeners deformed in the same direction in each plate panel.

Longitudinal stresses cause a plate to deflect with crests of the buckled shape perpendicular to the length of the plate, whereas shear stresses cause a plate to deflect with crests diagonally across the plate. Therefore, for each combination of compressive stress and shear stress, several finite element analyses were made, each with an orientation of crest of the initial imperfection differing by 5º or 10º to ensure that the near minimum failure load was identified.

Figure 3 shows the form of the initial imperfection for one half of the plate for the crest of the imperfection at 45º from the transverse stiffener. NSET = PLATE_0 shows the nodes along the longitudinal edge of the plate, NSET = PLATE_6 shows the nodes along the centre-line and the other lines are nodes at 12th points between the edge and the centre-line.

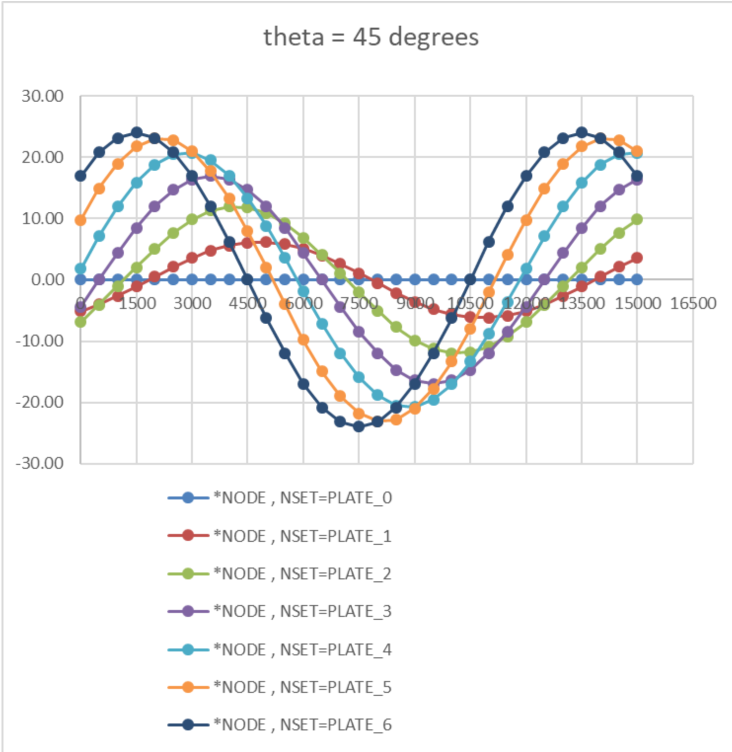

Figure 3 Initial imperfections for angle = 45^o

Figure 4 shows the initial imperfection of the transverse stiffener for the crest of the imperfection at 45^o from the transverse stiffener.

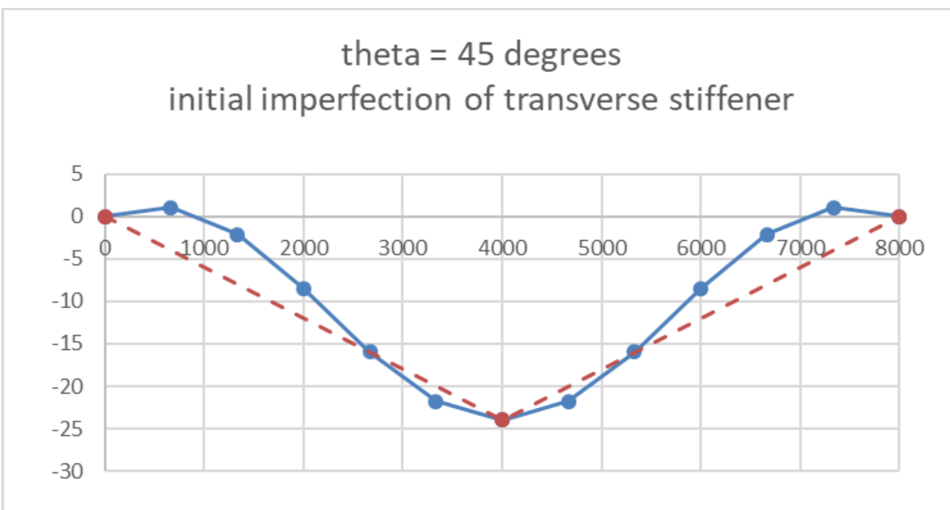

Figure 4 Initial imperfections along the line of the transverse stiffener for 45^o angle

The form of the imperfection was a half-sine curve across the width of the plate and a sine curve along the plate with length of the half-sine curve = 2a. The geometry of longitudinal sine curve was arranged to make the transverse crest skewed across the width of the plate. Defining the coordinates from one corner of the plate, with X as the longitudinal axis, Y as the transverse axis and Zas the out-of-plane axis, the imperfection is given by:

$$z = \delta_0 Sin\left[\frac{\pi\left(x + \frac{a}{2} - \left(\frac{b}{2} - y\right) Tan\varphi\right)}{2a}\right] Sin\left(\frac{\pi y}{b}\right) \quad (9)$$

where

δ_0 = the maximum initial imperfection, the lesser of $b_{sp}/250$ and 2a/250,

φ = the angle of the crest of the imperfection from the transverse stiffener.

The effect of the angle of inclination of the crest is shown in Figure 5 for two different ratios of axial and shear.

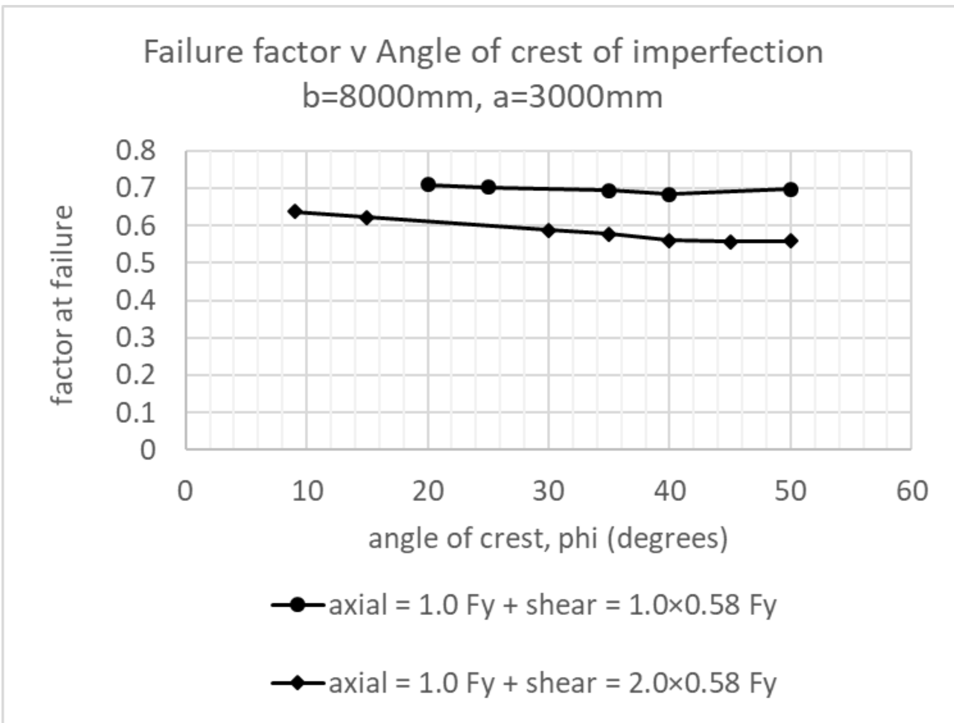

Figure 5 Effect of angle of crest of initial imperfections

6. Simplified analysis

6.1 Introduction to the incremental work method

Bridge designers have to process a very large number of load combinations on many variations of plate and stiffener sizes and spacings, so it is highly desirable to have a simplified analysis process. There is little time or resource available for finite element analysis in the design-build environment of modern bridge construction.

One possible process that is simple enough to apply through a spreadsheet is the calculation of incremental work that is described below. The author has implemented the calculation in only 70 rows, and it can probably be reduced from that. The method finds the ratio of destabilizing work to the stabilizing work for stiffened plate with maximum compression capacity and compares this with the same ratio for the applied compression plus coexistent shear. If the ratio for compression plus shear is not greater than the ratio for compression alone, then the stiffened plate is stable and may be designed as if it is loaded with compression alone equal to the maximum compression. This enables the factor C_{fv} to be calculated. This method has only been tested on the limited number of panels and stress combinations shown in this paper. Therefore, it would be wise to calibrate the method by at least one finite element analysis if applying it to plates of very different geometries. In this way, very few finite element analyses can used to cover many different cases through the spreadsheet calculations.

The work equations are written assuming that the maximum out of plane displacement of the transverse stiffener equals the nominal design values of initial imperfection and increase in imperfection. The equations in AASHTO LRFD 9th edition come from White et al which gives these as bsp/250 and bsp/370, where bsp is the overall width of the stiffened plate.

Finite element analysis shows significant plastic strains at failure of the stiffened plates. Therefore, the work is calculated as if it were a plastic mechanism assuming hinge lines along the lines labelled as "crest" and "trough" in Figure 6. The shape of the mechanism along the crest is defined by the triangular diagram "displacements along crest" reducing to zero displacements at the line "zero". This is an approximation, but Figure 4 shows that this is reasonable. The shape of the mechanism perpendicular to the crest is defined by the triangular diagram "maximum displacement" at the middle of the plate reducing to zero displacements along the edges of the plate. The maximum displacement, e, is equal to bsp/250 + bsp/370. The destabilizing work and stabilizing work are calculated assuming a unit change in displacement at the middle of the crest and that the shape of the displacements remains triangular.

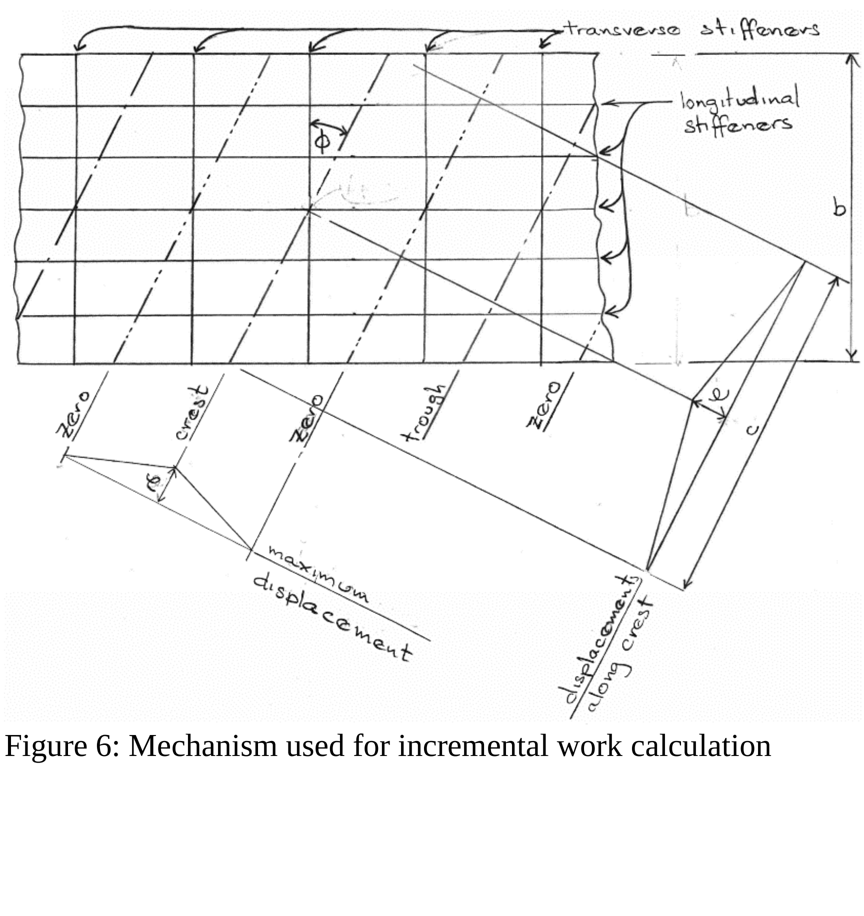

Figure 6: Mechanism used for incremental work calculation

The plate and the longitudinal stiffeners are assumed to be fully stressed by carrying the longitudinal compression and the in-plane shear, so they are assumed to not contribute to the stabilizing work. Therefore, the stabilizing work is done only by the transverse stiffeners. The stresses and deformations calculated by finite element analysis show that the majority of the stabilizing work is done by the transverse stiffener through the apex of the deformed shape, so the work model has reduction factors applied to the work calculated in the adjacent stiffeners.

6.2 Calculation of the destabilizing work

The destabilizing work comes from the displacement of the crest. The stresses in the plate perpendicular to the crest give a destabilizing component of force along the crest which is slightly reduced by the stresses in the plate parallel with the crest.

The maximum reaction/unit length, w_D, caused by the change of angle of the plate along the crest is

$$w_D = S_{cr} \times t \times 2 \times \frac{e}{a} \tag{10}$$

S_{cr} = the stress in the plate perpendicular to the crest, resolved from the shear and compression
t = the thickness of the plate
e = the displacement of the plate at the apex of the crest defined in Figure 6
a = the spacing of the transverse stiffeners

The maximum reaction/unit length, w_{Dpar}, caused by the change of angle of the plate perpendicular to the crest is

$$w_{Dpar} = S_{par} \times t \times 2 \times \frac{e}{(c/2)} \tag{11}$$

S_{cr} = the stress in the plate perpendicular to the crest, resolved from the shear and compression
c = the length of the crest between edges of the stiffened plate

The destabilizing work done by the stresses perpendicular to the crest is

$$w_D \times \frac{c}{3} \tag{12}$$

and the stabilizing work done by the stresses parallel with the crest is

$$w_{Dpar} \times \frac{2aCos\varphi}{3} \tag{13}$$

giving the net destabilizing work in the plate as

$$pwp = w_D \times \frac{c}{3} + w_{Dpar} \times \frac{2aCos\varphi}{3} \tag{14}$$

The longitudinal stiffeners also contribute to the destabilizing work by their reaction caused by their change of angle where they cross the crest. The destabilizing work done by the stiffeners is

$$pws = \sum S_x \times A_o \times 2 \times \frac{e_i}{a} \tag{15}$$

where
S_x = the longitudinal compressive stress in the stiffened plate
A_o = the area of the longitudinal stiffener, not including any plate
e_i = the displacement of the stiffener at the "hinge" on the crest defined in Figure 6

The total destabilizing work done is

$$dpw = pwp + pws \tag{16}$$

6.3 Calculation of stabilizing work

The stabilizing work arises from the bending moment at the "hinge" in the transverse stiffeners multiplied by the change of angle of the transverse stiffener at the "hinge".

For the stiffener through the apex of the deformed shape, the rotation is

$$\theta = 2 \times \frac{e}{(b/2)} \tag{17}$$

In cases where the crest crosses an adjacent stiffener, the rotation of each "hinge" is calculated assuming zero displacement of the transverse stiffener at the centre-line of the plate. It can be seen from the stresses and deformations in the finite element analysis that the contribution of the adjacent stiffeners is small. This is not found directly from the straight-line mechanism used for the work equations, so the displacement at each "hinge" is adjusted by a calibration with the finite element analysis, so the displacement of hinge "j" is taken as

$$e_{jc} = \left(\frac{e_j}{e}\right)^{p_c} \tag{18}$$

where
e_{jc}= the value of displacement at hinge "j" reduced by the calibration coefficient p_c
p_c = calibration coefficient found by comparing with finite element results, initial studies suggest the value of $p_c = 3.2$
e_j= the value of displacement at hinge "j" derived from the triangular displacements in Figure 6.

The rotation in the transverse stiffener nearest to the crest but not crossed by the crest is calculated using the displacement calculated for the quarter-point of the stiffener from the triangular displacement diagram in Figure 6 and assuming zero displacement of the transverse stiffener at the centre-line of the plate.

The total stabilizing work done is the sum of all the moments × rotations

$$spw = \sum M_r \times \theta_j \tag{19}$$

The only significant contribution from the other transverse stiffeners has been found to come from the stiffeners closest to the stiffener through the apex. These two stiffeners add a total of close to 25% to the work done by the stiffener though the apex.
The method uses a comparison of ratios of destabilizing work to stabilizing work for the case with shear compared to the maximum compression resistance,

$$\frac{dpw_i/spw_i}{dpw_{cr}/spw_{cr}} \tag{20}$$

where
dpw_i/spw_i= the ratio of destabilizing work to stabilizing work for load case i
dpw_{cr}/spw_{cr} = the ratio of destabilizing work to stabilizing work for the maximum compression resistance
so the magnitude of the moment of resistance, Mr, cancels from the calculation. Therefore, any convenient number may be used, provided this number is used throughout the calculations.

6.4 Application of the method

The method allows extrapolation from the finite element analyses. It is clear from Figure 7 that the method performs well for plates with a ratio of spacing of transverse stiffeners, a, to breadth, b, of a/b = 3/8. This ratio is representative of the spacing used in large bridge towers. For slightly different spacings, the method can be used to calculate values of Cfv. As the ratio a/b approaches 6/8, we can see that the method needs checking at higher shear values and a different calibration coefficient might be needed.

The steps of the method are as follows:

Step 1 is to calculate the compression resistance of the proposed stiffened plate and to size the transverse stiffeners required for the maximum compression resistance using the specified design code, for example AASHTO LRFD 9th edition.

Step 2 is to apply the incremental work method to the case of longitudinal compression resistance in Step 1, and find the ratio of destabilizing work to stabilizing work, dpw/spw, for the crest along the line of a transverse stiffener, so the angle $\phi = 0$.

Step 3 is to apply the method to the stiffened plate with compression and coincident shear, varying the angle between the transverse stiffener and the crest, ϕ, to find the maximum ratio of destabilizing to stabilizing work. This maximum ratio is compared with the ratio from Step 2. If the ratio from Step 3 is not greater than the ration from Step 2, the stiffened plate is stable.

Step 4 is to calculate Cfv from the compression and shear values found in Step 3.

$$C_{fv} = \frac{(\sigma_{1r} - \sigma_{1fve})}{f_{ve}} \tag{21}$$

where

σ_{1r} = longitudinal stress resistance of the stiffened plate without shear
σ_{1fve} = longitudinal stress at failure of the stiffened plate with coexistent in-plane shear calculated by the method
f_{ve} = the coexistent shear stress in the stiffened plate

6.5 Comparison of the method with finite element analysis results

The method is compared with finite element analysis results in Figure 7. It is important to remember that the method calculates only the destabilizing effect of shear. It does not check the capacity the plate or of the transverse stiffener which must be done using F_{tr} = limiting value of transverse stress in the plate, reduced below the nominal yield stress, as described in Section 2. The plots show results for the panels described in Table 1, in which the yield stress of the longitudinal stiffeners is 345 MPa (50 ksi), but the yield stress of the plate was 2000 MPa to uncouple the destabilizing effect of shear from the weakening effect of the yield criterion.

The work method performs well for the 3m spacing of transverse stiffeners, becoming conservative at higher shear forces. The method is very conservative at low shear for the 6m spacing, but is less conservative for higher shears.

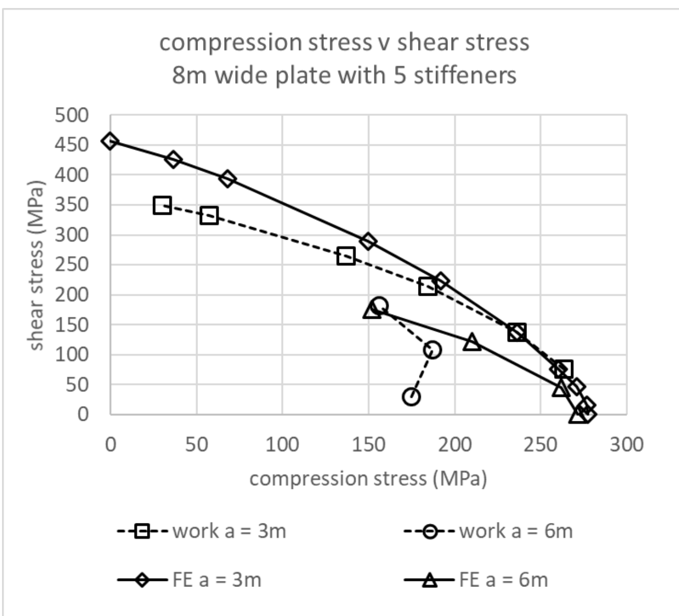

Figure 7: Mechanism used for incremental work calculation

Conclusion

Shear forces in stiffened plates increase the destabilizing effects on the transverse stiffeners. This is caused both by (a) the reduction of bending resistance caused by the yield criteria in the plate and by (b) the out of plane forces generated by the in-plane stress field acting on the initial geometrical imperfections in the stiffened plate. Finite element studies show that small shear forces have a very small effect, confirming that code design of transverse stiffeners for longitudinal compression are satisfactory even where there is coincident shear, provided that the net shear force is small. At high values of net shear force, the effect of the shear force is disproportionately higher.

For design, the effect of the shear force on the transverse stiffeners can be evaluated by adding a proportion of the shear to the longitudinal compression to give an equivalent longitudinal compression.

Alternatively, the destabilizing effect of shear on the transverse stiffeners can be evaluated by the work equation method which compares the destabilizing effect of shear plus compression with the destabilizing effect of the compression resistance of the stiffened plate. This method relies on calibration with finite element analysis. The capacity of the transverse stiffener must finally be checked by the equations in Section 2 to account for the yield criterion. The method appears to perform reasonably for closely spaced transverse stiffeners, as used in large bridge towers, but is conservative for widely spaced stiffeners.

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